Review Problems
for Fluid Mechanics

Review Problems for Fluid Mechanics

Edited by The Department of Mechanics

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Chapter 1 Introduction and Properties of Fluids

1-1.

Determine the change in the density of oxygen when the absolute pressure changes from 345 kPa to 286 kPa, while the temperature *remains constant* at 25°C. This is called an *isothermal process*.

SOLUTION

Applying the ideal gas law with $T_1 = (25°\text{C} + 273)\ \text{K} = 298\ \text{K}$, $p_1 = 345\ \text{kPa}$ and $R = 259.8\ \text{J/kg}\cdot\text{K}$ for oxygen (table in Appendix A),

$$p_1 = \rho_1 R T_1; \qquad 345(10^3)\ \text{N/m}^2 = \rho_1(259.8\ \text{J/kg}\cdot\text{K})(298\ \text{K})$$
$$\rho_1 = 4.4562\ \text{kg/m}^3$$

For $p_2 = 286\ \text{kPa}$ and $T_2 = T_1 = 298\ \text{K}$,

$$p_2 = \rho_2 R T_2; \qquad 286(10^3)\ \text{N/m}^2 = \rho_2(259.8\ \text{J/kg}\cdot\text{K})(298\ \text{K})$$
$$\rho_2 = 3.6941\ \text{kg/m}^3$$

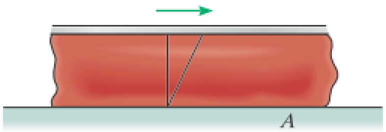
Thus, the change in density is

$$\Delta\rho = \rho_2 - \rho_1 = 3.6941\ \text{kg/m}^3\ \ -4.4562\ \text{kg/m}^3$$
$$= -0.7621\ \text{kg/m}^3\ \ - 0.762\ \ \text{kg/m}^3 \qquad \textbf{Ans.}$$

The negative sign indicates a decrease in density

1-2.

An experimental test using human blood at $T = 30°C$ indicates that it exerts a shear stress of $\tau = 0.15\ \text{N/m}^2$ on surface A, where the measured velocity gradient is $16.8\ \text{s}^{-1}$. Since blood is a non-Newtonian fluid, determine its *apparent viscosity* at A.



SOLUTION

Here, $\dfrac{du}{dy} = 16.8\ \text{s}^{-1}$ and $\tau = 0.15\ \text{N/m}^2$. Thus,

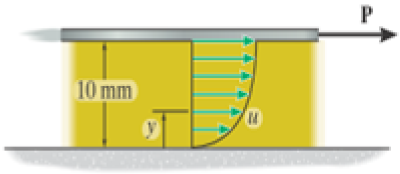
$$\tau = \mu_a \frac{du}{dy}; \qquad 0.15\ \text{N/m}^2 = \mu_a\left(16.8\ \text{s}^{-1}\right)$$

$$\mu_a = 8.93\left(10^{-3}\right)\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Realize that blood is a non-Newtonian fluid. For this reason, we are calculating the *apparent* viscosity.

1-3.

When the force **P** is applied to the plate, the velocity profile for a Newtonian fluid that is confined under the plate is approximated by $u = (4.23y^{1/3})$ mm/s, where y is in mm. Determine the shear stress within the fluid at $y = 5$ mm. Take $\mu = 0.630(10^{-3})\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = (4.23y^{1/3})\ \mathrm{mm/s}$$

$$\frac{du}{dy} = \left[\frac{1}{3}(4.23)y^{-2/3}\right]\ \mathrm{s^{-1}}$$

$$= \left(\frac{1.41}{y^{2/3}}\right)\ \mathrm{s^{-1}}$$

At $y = 5$ mm,

$$\frac{du}{dy} = \left(\frac{1.41}{5^{2/3}}\right)^{-1} = 0.4822\ \mathrm{s^{-1}}$$

The shear stress is

$$\tau = \mu\frac{du}{dy} = [0.630(10^{-3})\ \mathrm{N \cdot s/m^2}]\ 0.4822\ \mathrm{s^{-1}}$$

$$= 0.3038(10^{-3})\ \mathrm{N/m^2}$$

$$= 0.304\ \mathrm{mPa}$$

Note: When $y = 0$, $\frac{du}{dy} \rightarrow \infty$ and so $\tau \rightarrow \infty$. at this point.

Hence, The equation cannot be applied at this point.

1-4.

If the kinematic viscosity of glycerin is $\nu = 1.15(10^{-3})\ \text{m}^2/\text{s}$, determine its viscosity in FPS units. At the temperature considered, glycerin has a specific gravity of $S_g = 1.26$.

SOLUTION

The density of glycerin is

$$\rho_g = S_g \rho_w = 1.26\left(1000\ \text{kg/m}^3\right) = 1260\ \text{kg/m}^3$$

Then,

$$\nu_g = \frac{\mu_y}{\rho_g}; \quad 1.15\left(10^{-3}\right)\ \text{m}^2/\text{s} = \frac{\mu_g}{1260\ \text{kg/m}^3}$$

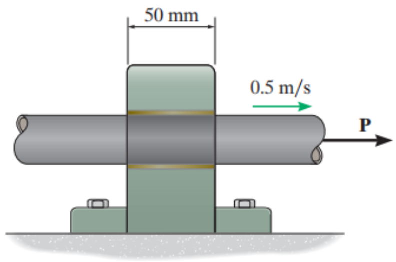
$$\mu_g = \left(1.449\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2$$

$$= 0.03026\ \text{lb}\cdot\text{s/ft}^2$$

$$= 0.0303\ \text{lb}\cdot\text{s/ft}^2 \qquad \textbf{Ans.}$$

1-5.

If a force of $P = 2$ N causes the 30-mm-diameter shaft to slide along the lubricated bearing with a constant speed of 0.5 m/s, determine the viscosity of the lubricant and the constant speed of the shaft when $P = 8$ N. Assume the lubricant is a Newtonian fluid and the velocity profile between the shaft and the bearing is linear. The gap between the bearing and the shaft is 1 mm.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{2\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = \mu\left(\frac{0.5\text{ m/s}}{0.001\text{ m}}\right)$$

$$\mu = 0.8498\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Thus,

$$\frac{8\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = (0.8488\text{ N}\cdot\text{s/m}^2)\left(\frac{v}{0.001\text{ m}}\right)$$

$$v = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

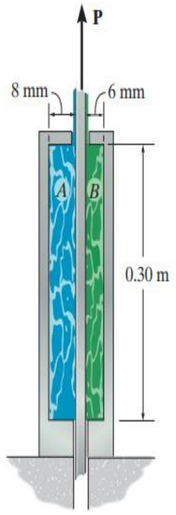
Also, by proportion,

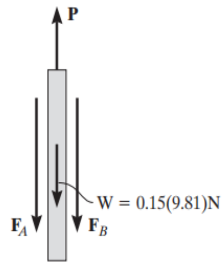
$$\frac{\left(\frac{2\text{ N}}{A}\right)}{\left(\frac{8\text{ N}}{A}\right)} = \frac{\mu\left(\frac{0.5\text{ m/s}}{t}\right)}{\mu\left(\frac{v}{t}\right)}$$

$$v = \frac{4}{2}\text{ m/s} = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

1-6.

A plastic strip having a width of 0.2 m and a mass of 150 g passes between two layers A and B of paint having a viscosity of $5.24 \text{ N}\cdot\text{s/m}^2$. Determine the force **P** required to overcome the viscous friction on each side if the strip moves upwards at a constant speed of 4 mm/s. Neglect any friction at the top and bottom openings, and assume the velocity profile through each layer is linear.





(a)

SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant. For layers A and B,

$$\left(\frac{du}{dy}\right)_A = \frac{4 \text{ mm/s}}{8 \text{ mm}} = 0.5 \text{ s}^{-1} \quad \left(\frac{du}{dy}\right)_B = \frac{4 \text{ mm/s}}{6 \text{ mm}} = 0.66675 \text{ s}^{-1}$$

The shear stresses acting on the surfaces in contact with layers A and B are

$$\tau_A = \mu\left(\frac{du}{dy}\right)_A = (5.24 \text{ N}\cdot\text{s/m}^2)(0.5 \text{ s}^{-1}) = 2.62 \text{ N/m}^2$$

$$\tau_B = \mu\left(\frac{du}{dy}\right)_B = (5.24 \text{ N}\cdot\text{s/m}^2)(0.6667 \text{ s}^{-1}) = 3.4933 \text{ N/m}^2$$

Thus, the shear forces acting on the contact surfaces are

$$F_A = \tau_A A = (2.62 \text{ N/m}^2)[(0.2 \text{ m})(0.3 \text{ m})] = 0.1572 \text{ N}$$

$$F_B = \tau_B A = (3.4933 \text{ N/m}^2)[(0.2 \text{ m})(0.3 \text{ m})] = 0.2096 \text{ N}$$

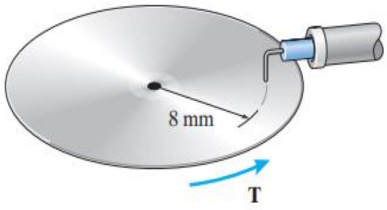
Consider the force equilibrium along y axis for the FBD of the strip, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad P - 0.15(9.81) \text{ N} - 0.1572 \text{ N} - 0.2096 \text{ N} = 0$$

$$P = 1.8383 \text{ N} = 1.84 \text{ N}$$ **Ans.**

1-7.

The read–write head for a hand-held music player has a surface area of 0.04 mm^2. The head is held 0.04 μm above the disk, which is rotating at a constant rate of 1800 rpm. Determine the torque **T** that must be applied to the disk to overcome the frictional shear resistance of the air between the head and the disk. The surrounding air is at standard atmospheric pressure and a temperature of 20°C. Assume the velocity profile is linear.



SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant. For layers A and B,

$$\left(\frac{du}{dy}\right)_A = \frac{4\text{ mm/s}}{8\text{ mm}} = 0.5\text{ s}^{-1} \quad \left(\frac{du}{dy}\right)_B = \frac{4\text{ mm/s}}{6\text{ mm}} = 0.66675\text{ s}^{-1}$$

The shear stresses acting on the surfaces in contact with layers A and B are

$$\tau_A = \mu\left(\frac{du}{dy}\right)_A = (5.24\text{ N}\cdot\text{s/m}^2)(0.5\text{ s}^{-1}) = 2.62\text{ N/m}^2$$

$$\tau_B = \mu\left(\frac{du}{dy}\right)_B = (5.24\text{ N}\cdot\text{s/m}^2)(0.6667\text{ s}^{-1}) = 3.4933\text{ N/m}^2$$

Thus, the shear forces acting on the contact surfaces are

$$F_A = \tau_A A = (2.62\text{ N/m}^2)[(0.2\text{ m})(0.3\text{ m})] = 0.1572\text{ N}$$

$$F_B = \tau_B A = (3.4933\text{ N/m}^2)[(0.2\text{ m})(0.3\text{ m})] = 0.2096\text{ N}$$

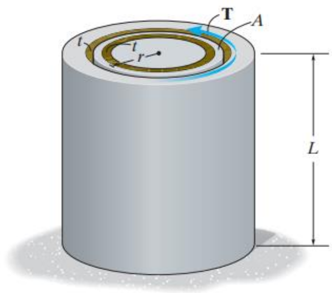
Consider the force equilibrium along y axis for the FBD of the strip, Fig. a.

$$+\uparrow\Sigma F_y = 0; \qquad P - 0.15(9.81)\text{ N} - 0.1572\text{ N} - 0.2096\text{ N} = 0$$

$$P = 1.8383\text{ N} = 1.84\text{ N}$$ **Ans.**

1-8.

The very thin tube A of mean radius r and length L is placed within the fixed circular cavity as shown. If the cavity has a small gap of thickness t on each side of the tube, and is filled with a Newtonian liquid having a viscosity μ, determine the torque **T** required to overcome the fluid resistance and rotate the tube with a constant angular velocity of ω. Assume the velocity profile within the liquid is linear.

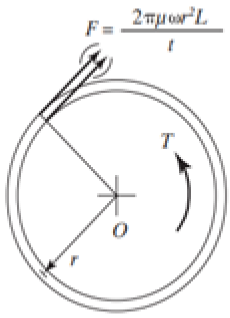


SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{(\omega r)}{t}$$



(a)

Considering the moment equilibrium of the tube, Fig. a,

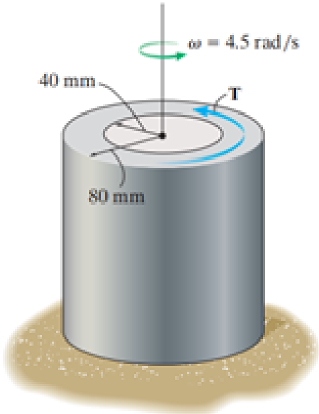
$$\Sigma M = 0; \qquad T - 2\tau A r = 0$$

$$T = 2(\mu)\frac{(\omega r)}{t}(2\pi r L)r$$

$$T = \frac{4\pi\mu\omega r^3 L}{t}$$ **Ans.**

1-9.

The tube rests on a 1.5-mm thin film of oil having a viscosity of $\mu = 0.0586\ \mathrm{N\cdot s/m^2}$. If the tube is rotating at a constant angular velocity of $\omega = 4.5\ \mathrm{rad/s}$, determine the torque **T** that must be applied to the tube to maintain the motion. Assume the velocity profile within the oil is linear.



SOLUTION

Oil is a Newtonian fluid. Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus, $\frac{du}{dy} = \frac{U}{t} = \frac{wr}{t}$.

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

Thus, the shear force the oil exerts on the differential element of area $dA = 2\pi r\, dr$ shown shaded in Fig. *a* is

$$dF = \tau dA = \left(\frac{\mu \omega r}{t}\right)(2\pi r\, dr) = \frac{2\pi \mu \omega}{t} r^2 dr$$

Considering the moment equilibrium of the tube about point D,

$$\circlearrowleft + \Sigma M_O = 0; \qquad \int_{r_i}^{r_o} r dF - T = 0$$

$$T = \int_{r_i}^{r_o} r dF = \frac{2\pi \mu \omega}{t} \int_{r_i}^{r_o} r^3 dr$$

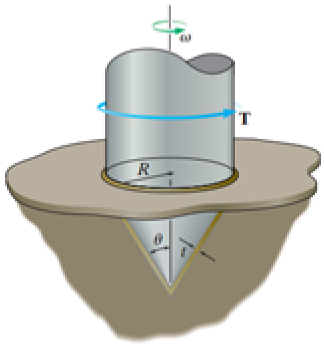
$$= \frac{2\pi \mu \omega}{t} \left(\frac{r^4}{4}\right)\Bigg|_{r_i}^{r_o} = \frac{\pi \mu \omega}{2t}(r_o^4 - r_i^4)$$

Substituting the numerical values,

$$T = \frac{\pi(0.0586\ \mathrm{N\cdot s/m^2})(4.5\ \mathrm{rad/s})}{2[1.5\ (10^{-3})\ \mathrm{m}]}(0.08^4 - 0.04^4)$$

$$= 0.01060\ \mathrm{N\cdot m} = 0.0106\ \mathrm{N\cdot m}$$ **Ans.**

1-10.



The conical bearing is placed in a lubricating Newtonian fluid having a viscosity μ. Determine the torque **T** required to rotate the bearing with a constant angular velocity of ω. Assume the velocity profile along the thickness t of the fluid is linear.

SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus,

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

From the geometry shown in Fig. a,

$$z = \frac{r}{\tan\theta} \qquad dz = \frac{dr}{\tan\theta} \qquad \textbf{(1)}$$

Also, from the geometry shown in Fig. b,

$$dz = ds\cos\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{dr}{\tan\theta} = ds\cos\theta \qquad ds = \frac{dr}{\sin\theta}$$

The area of the surface of the differential element shown shaded in Fig. a is $dA = 2\pi r ds = \dfrac{2\pi}{\sin\theta} r dr$. Thus, the shear force the oil exerts on this area is

$$dF = \tau dA = \left(\frac{\mu\omega r}{t}\right)\left(\frac{2\pi}{\sin\theta} r dr\right) = \frac{2\pi\mu\omega}{t\sin\theta} r^2 dr$$

Considering the moment equilibrium of the shaft, Fig. a,

$$\Sigma M_z = 0; \qquad T - \int r dF = 0$$

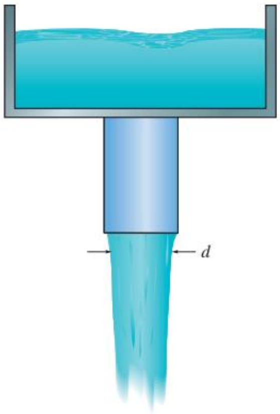
$$T = \int r dF = \frac{2\pi\mu\omega}{t\sin\theta}\int_0^R r^3 dr$$

$$= \frac{2\pi\mu\omega}{t\sin\theta}\left(\frac{r^4}{4}\right)\Big|_0^R$$

$$= \frac{\pi\mu\omega R^4}{2t\sin\theta} \qquad \textbf{Ans.}$$

1-11.

For water falling out of the tube, there is a difference in pressure Δp between a point located just inside and a point just outside of the stream due to the effect of surface tension σ. Determine the diameter d of the stream at this location.



SOLUTION

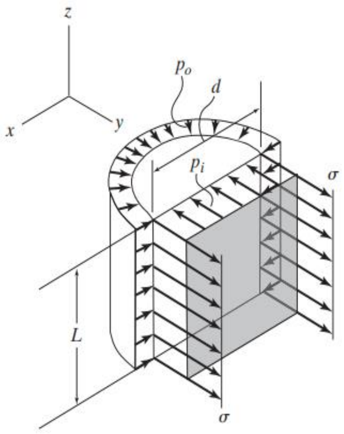
Consider a length L of the water column. The free-body diagram of half column is shown in Fig. *a*. Consider the force equilibrium along the *y*-axis,

$$\Sigma F_y = 0; \quad 2\sigma L + p_o[d(L)] - p_i[d(L)] = 0$$

$$2\sigma = (p_i - p_o)d$$

However, $p_i - p_o = \Delta p$. Then

$$d = \frac{2\sigma}{\Delta p}$$

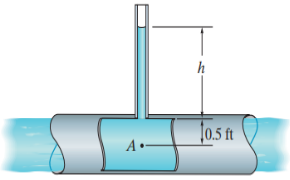


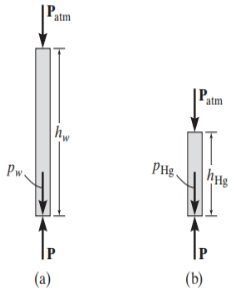
(a)

Chapter 2 Fluid Statics

2-1.

If the piezometer measures a gage pressure of 10 psi at point A, determine the height h of the water in the tube. Compare this height with that using mercury. Take $\rho_w = 1.94\ \text{slug/ft}^3$ and $\rho_{Hg} = 26.3\ \text{slug/ft}^3$.





SOLUTION

Here, the absolute pressure to be measured is

$$p = p_g + p_{atm} = \left(10\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + p_{atm} = (1440 + p_{atm})\ \frac{\text{lb}}{\text{ft}^2}$$

For the water piezometer, Fig. *a*,

$$p = p_{atm} + p_w; \quad (1440 + p_{atm})\ \frac{\text{lb}}{\text{ft}^3} = p_{atm} + (1.94\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(h + 0.5\ \text{ft})$$

$$h_w = 22.55\ \text{ft} = 22.6\ \text{ft} \qquad \textbf{Ans.}$$

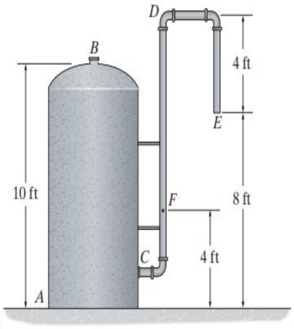
For the mercury piezometer, Fig. *b*,

$$p = p_{atm} + p_{Hg}; \quad (1440 + p_{atm})\ \frac{\text{lb}}{\text{ft}^3} = p_{atm} + (26.3\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(h + 0.5\ \text{ft})$$

$$h_{Hg} = 1.20\ \text{ft} \qquad \textbf{Ans.}$$

2-2.

The field storage tank is filled with oil. The standpipe is connected to the tank at C, and the system is open to the atmosphere at B and E. Determine the maximum pressure in the tank in psi if the oil reaches a level of F in the pipe. Also, at what level should the oil be in the tank, so that the maximum pressure occurs in the tank? What is this value? Take $\rho_o = 1.78\ \text{slug/ft}^3$.



SOLUTION

Since the top of the tank is open to the atmosphere, the free surface of the oil in the tank will be the same height as that of point F. Thus, the maximum pressure which occurs at the base of the tank (level A) is

$$(p_A)_g = \gamma h$$

$$= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4\ \text{ft})$$

$$= 229.26\ \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.59\ \text{psi} \qquad \textbf{Ans.}$$

Absolute maximum pressure occurs at the base of the tank (level A) when the **oil reaches level B.**

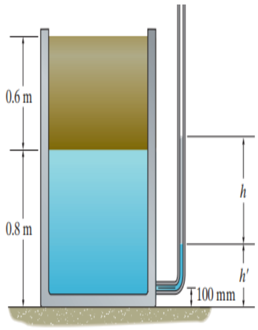
$$(p_A)_{\substack{\text{abs}\\ \text{max}}} = \gamma h$$

$$= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(10\ \text{ft})$$

$$= 573.16\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 3.98\ \text{psi} \quad \textbf{Ans.}$$

2-3.

Determine the level h' of water in the tube if the depths of oil and water in the tank are 0.6 m and 0.8 m, respectively, and the height of mercury in the tube is $h = 0.08$ m. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, and $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



SOLUTION

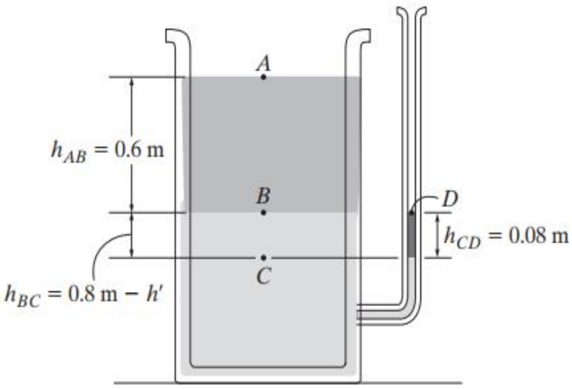
Referring to Fig. a, $h_{AB} = 0.6$ m, $h_{BC} = 0.8 - h'$ and $h_{CD} = h = 0.08$ m. Then the manometer rule gives

$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{Hg} g h_{CD} = p_D$$

Here, $p_A = p_D = 0$, since points A and D are exposed to the atmosphere.

$$0 + (900\ \text{kg/m}^3)(g)(0.6\ \text{m}) + (1000\ \text{kg/m}^3)(g)(0.8\ \text{m} - h')$$
$$-(13550\ \text{kg/m}^3)(g)(0.08\ \text{m}) = 0$$
$$h' = 0.256\ \text{m} = 256\ \text{mm} \qquad \textbf{Ans.}$$

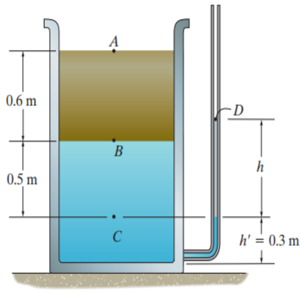
Note: Since $0.1\ \text{m} < h' < 0.8\ \text{m}$, the solution is **OK**!



(a)

2-4.

Determine the height h of the mercury in the tube if the level of water in the tube is $h' = 0.3$ m and the depths of the oil and water in the tank are 0.6 and 0.5 m, respectively. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, and $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



SOLUTION

Referring to Fig. a, $h_{AB} = 0.6\ \text{m}$, $h_{BC} = 0.8\ \text{m} - 0.3\ \text{m} = 0.5\ \text{m}$ and $h_{CD} = h$. Then the manometer rule gives

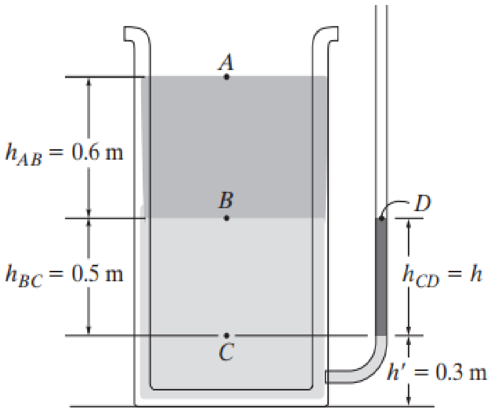
$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{Hg} g h_{CD} = p_D$$

Here, $p_A = p_D = 0$, since points A and D are exposed to the atmosphere.

$$0 + (900\ \text{kg/m}^3)(g)(0.6\ \text{m}) + (1000\ \text{kg/m}^3)(g)(0.5\ \text{m})$$

$$-(13550\ \text{kg/m}^3)(g)(h) = 0$$

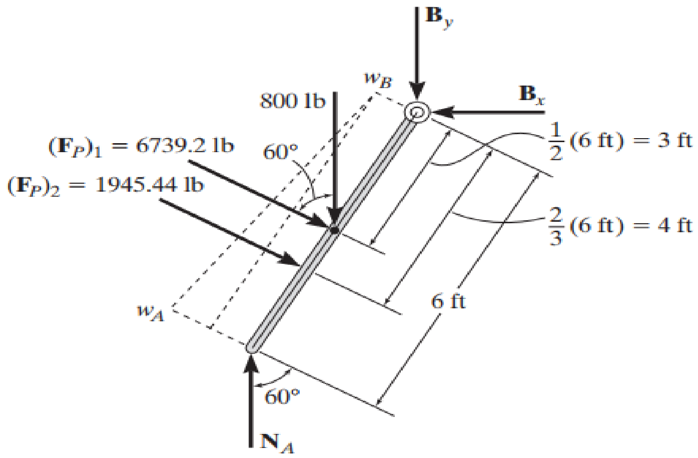
$$h = 0.07675\,\text{m} = 76.8\ \text{mm} \qquad \textbf{Ans.}$$



(a)

2-5.

The uniform rectangular relief gate AB has a weight of 800 lb and a width of 2 ft. Determine the components of reaction at the pin B and the normal reaction at the smooth support A.



(a)

SOLUTION

Here, $h_B = 9$ ft and $h_A = 9\text{ ft} + 6\text{ ft}\sin 60° = 14.20$ ft. Thus, the intensities of the distributed load at B and A are

$$w_B = \gamma_w h_B b = (62.4\text{ lb/ft}^3)(9\text{ ft})(2\text{ ft}) = 1123.2\text{ lb/ft}$$
$$w_A = \gamma_w h_A b = (62.4\text{ lb/ft}^3)(14.20\text{ ft})(2\text{ ft}) = 1771.68\text{ lb/ft}$$

Thus,

$$(F_p)_1 = (1123.2\text{ lb/ft})(6\text{ ft}) = 6739.2\text{ lb}$$

$$(F_p)_2 = \frac{1}{2}(1771.68\text{ lb/ft} - 1123.2\text{ lb/ft})(6\text{ ft}) = 1945.44\text{ lb}$$

Write the moment equation of equilibrium about B by referring to the FBD of the gate, Fig. *a*.

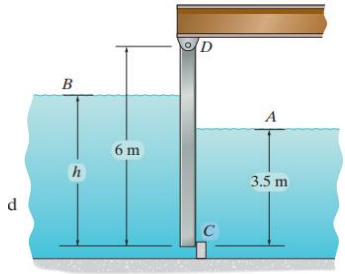
$$\circlearrowleft + \Sigma M_B = 0; \quad (800\text{ lb})\cos 60°(3\text{ ft}) + (6739.2\text{ lb})(3\text{ ft}) + (1945.44\text{ lb})(4\text{ ft}) - N_A\cos 60°(6\text{ ft}) = 0$$
$$N_A = 9733.12\text{ lb} = 9.73\text{ kip} \qquad \textbf{Ans.}$$

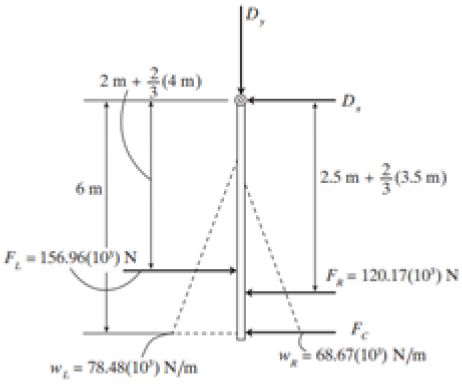
Using this result to write the force equations of equilibrium along x and y axes,

$$\overset{+}{\leftarrow}\Sigma F_x = 0; \quad B_x - (6739.2\text{ lb})\sin 60° - (1945.44\text{ lb})\sin 60° = 0$$
$$B_x = 7521.12\text{ lb} = 7.52\text{ kip} \qquad \textbf{Ans.}$$
$$+\uparrow\Sigma F_y = 0; \quad 9733.12\text{ lb} - 800\text{ lb} - (6739.2\text{ lb})\cos 60° - (1945.44\text{ lb})\cos 60° - B_y = 0$$
$$B_y = 4590.80\text{ lb} = 4.59\text{ kip} \qquad \textbf{Ans.}$$

2-6.

The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. For the water level $h = 4$ m, determine the horizontal reaction at the smooth stop C. The gate has a width of 2 m. At what height h will the gate be on the verge of opening?





SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed load on the left and right sides of the gate at C are

$$(w_C)_L = \rho_w g h_{BC}(b) = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4\text{ m})(2\text{ m})$$
$$= 78.48\left(10^3\right)\text{ N/m}$$
$$(w_C)_R = \rho_w g h_{AC}(b) = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(3.5\text{ m})(2\text{ m})$$
$$= 68.67\left(10^3\right)\text{ N/m}$$

The resultant triangular distributed load on the left and right sides of the gate is shown on its free-body diagram, Fig. *a*,

$$F_L = \frac{1}{2}(w_C)_L L_{BC} = \frac{1}{2}\left(78.48\left(10^3\right)\text{ N/m}\right)(4\text{ m}) = 156.96\left(10^3\right)\text{ N}$$
$$F_R = \frac{1}{2}(w_C)_R L_{AC} = \frac{1}{2}\left(68.67\left(10^3\right)\text{ N/m}\right)(3.5\text{ m}) = 120.17\left(10^3\right)\text{ N}$$

These results can also be obtained as follows:

$$F_L = \gamma \bar{h}_L A_L = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(2\text{ m})\left[(4\text{ m})(2\text{ m})\right] = 156.96\left(10^3\right)\text{ N}$$
$$F_R = \gamma \bar{h}_R A_R = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.75\text{ m})\left[3.5\text{ m}(2\text{ m})\right] = 120.17\left(10^3\right)\text{ N}$$

Referring to the free-body diagram of the gate in Fig. *a*,

$$\circlearrowleft + \Sigma M_D = 0; \quad \left[156.96\left(10^3\right)\text{ N}\right]\left[2\text{ m} + \frac{2}{3}(4\text{ m})\right] - \left[120.17\left(10^3\right)\text{ N}\right]\left[2.5\text{ m} + \frac{2}{3}(3.5\text{ m})\right] - F_C(6\text{ m}) = 0$$

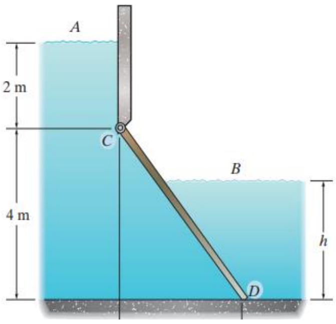
$$F_C = 25.27\left(10^3\right)\text{ N} = 25.3\text{ kN} \qquad \textbf{Ans.}$$

When $h = 3.5$ m, the water levels are equal. Since $F_C = 0$, the gate will open.

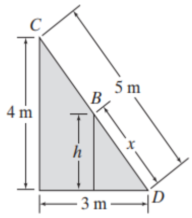
$$h = 3.5\text{ m} \qquad \textbf{Ans.}$$

2-7.

The uniform plate, which is hinged at C, is used to control the level of the water at A to maintain its constant depth of 6 m. If the plate has a width of 1.5 m and a mass of 30 Mg, determine the required minimum height h of the water at B so that seepage will not occur at D.



SOLUTION



(a)

Referring to the geometry shown in Fig. a,

$$\frac{x}{5} = \frac{h}{4}; \qquad x = \frac{5}{4}h$$

The intensities of the distributed load shown in the FBD of the ga

$$w_1 = \rho_w g h_1 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})(1.5\ \text{m}) = 29.4\ \text{N}$$
$$w_2 = \rho_w g h_2 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(1.5\ \text{m}) = 88.29(10^3)\ \text{N},$$
$$w_3 = \rho_w g h_3 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(1.5\ \text{m}) = [14.715(10^3)h]$$

Then, the resultant forces of these distributed loads are

$$F_1 = w_1 l_{CD} = [29.43(10^3)\ \text{N/m}](5\ \text{m}) = 147.15(10^3)\ \text{N}$$
$$F_2 = \frac{1}{2}(w_2 - w_1)l_{CD} = \frac{1}{2}[88.29(10^3)\ \text{N/m} - 2943(10^3)\ \text{N/m}](5\ \text{m}) = 1$$
$$F_3 = \frac{1}{2}w_3 l_{BD} = \frac{1}{2}[14.715(10^3)h]\left(\frac{5}{4}h\right) = [9.196875(10^3)h^2]\ \text{N}$$

and act at

$$d_1 = \frac{1}{2}(5\ \text{m}) = 2.5\ \text{m} \quad d_2 = \frac{2}{3}(5\ \text{m}) = 3.3333\ \text{m}$$
$$d_3 = 5\ \text{m} - \frac{1}{3}\left(\frac{5}{4}h\right) = (5 - 0.4167h)\ \text{m}$$

The seepage is on the verge of occurring when the gate is about to open. Thus, it is required that $N_D = 0$. Write the moment equation of equilibrium about point C by referring to Fig. a.

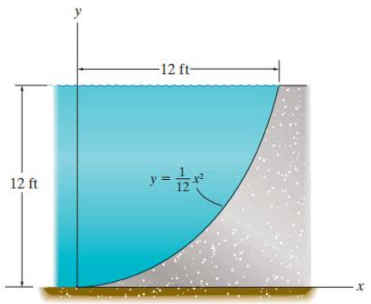
$$\circlearrowleft + \Sigma M_C = 0; \quad [147.15(10^3)\ \text{N}](2.5\ \text{m}) + [147.15(10^3)\ \text{N}](3.3333\ \text{m})$$
$$- [30(10^3)(9.81)\ \text{N}]\left(\frac{3}{5}\right)(2.5\ \text{m})$$
$$- [9.196875(10^3)h^2](5 - 0.4167h) = 0$$
$$3.8320h^3 - 45.9844h^2 + 416.925 = 0$$

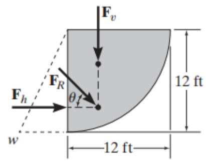
Solving numerically,

$$h = 3.5987\ \text{m} = 3.60\ \text{m}$$ **Ans.**

2-8.

The wall is in the form of a parabola. Determine the magnitude and direction of the resultant force on the wall if it is 8 ft wide.





(a)

SOLUTION

The horizontal loading on the wall is due to the pressure on the vertical projected area of the wall, Fig. *a*. Since the wall has a constant width of $b = 8$ ft, the intensity of the horizontal distributed load at the base of the wall is

$$w = \gamma_w hb = (62.4\ \text{lb/ft}^3)(12\ \text{ft})(8\ \text{ft}) = 5.9904(10^3)\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}wh = \frac{1}{2}[5.9904(10^3)\ \text{lb/ft}](12\ \text{ft}) = 35.9424(10^3)\text{lb}$$

The vertical force acting on the wall is equal to the weight of the water contained in the block above the wall (shown shaded in Fig. *a*). From the inside back cover of the text, the volume of this block (parabolic cross-section) is

$$\mathcal{V} = \frac{2}{3}ahb = \frac{2}{3}(12\ \text{ft})(12\ \text{ft})(8\ \text{ft}) = 768\ \text{ft}^3$$

Thus,

$$F_v = y_w\mathcal{V} = (62.4\ \text{lb/ft}^3)(768\ \text{ft}^3) = 47.9232(10^3)\ \text{lb}$$

Then the magnitude of the resultant force is

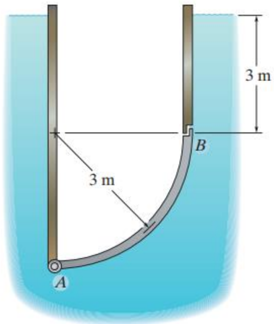
$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{[35.9424(10^3)\ \text{lb}]^2 + [47.9232(10^3)\ \text{lb}]^2}$$
$$= 59.904(10^3)\ \text{lb} = 59.9\ \text{kip} \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left[\frac{47.9232(10^3)\ \text{lb}}{35.9424(10^3)\ \text{lb}}\right] = 53.13^\circ = 53.1^\circ \qquad \textbf{Ans.}$$

2-9.

Determine the horizontal and vertical components of reaction at the hinge A and the horizontal normal reaction at B caused by the water pressure. The gate has a width of 3 m.



SOLUTION

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(6\ \text{m})(3\ \text{m}) = 176.58\left(10^3\right)\ \text{N/m}$$

$$w_B = \rho_w g h_B b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m})(3\ \text{m}) = 88.29\left(10^3\right)\ \text{N/m}$$

Thus,

$$(F_h)_1 = \left[88.29\left(10^3\right)\ \text{N/m}\right](3\ \text{m}) = 264.87\left(10^3\right)\ \text{N} = 264.87\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[176.58\left(10^3\right)\ \text{N/m} - 88.29\left(10^3\right)\ \text{N/m}\right](3\ \text{m}) = 132.435\left(10^3\right)\ \text{N} = 132.435\ \text{kN}$$

They act at

$$\tilde{y}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{y}_2 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

The vertical component of the resultant force acting on the gate is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*), but acts upward.

$$(F_v)_1 = \rho_w g \forall_1 = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(3\ \text{m})(3\ \text{m})(3\ \text{m})\right] = 264.87\left(10^3\right)\ \text{N} = 264.87\ \text{kN}$$

$$(F_v)_2 = \rho_w g \forall_2 = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[\frac{\pi}{4}(3\ \text{m})^2(3\ \text{m})\right] = 66.2175\pi\left(10^3\right)\ \text{N} = 66.2175\pi\ \text{kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{x}_2 = \frac{4(3\ \text{m})}{3\pi} = \left(\frac{4}{\pi}\right)\ \text{m}$$

Considering the equilibrium of the FBD of the gate in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad (264.87\text{ kN})(1.5\text{ m}) + (132.435\text{ kN})(1\text{ m}) + (264.87\text{ kN})(1.5\text{ m})$$
$$+ (66.2175\pi\text{ kN})\left(\frac{4}{\pi}\text{ m}\right) - N_B(3\text{ m}) = 0$$

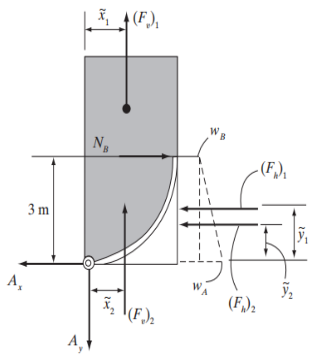
$$N_B = 397.305\text{ kN} = 397\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 397.305\text{ kN} - 264.87\text{ kN} - 132.435\text{ kN} - A_x = 0$$

$$A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 264.87\text{ kN} + 66.2175\pi\text{ kN} - A_y = 0$$

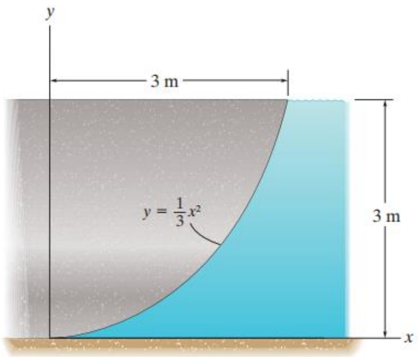
$$A_y = 472.90\text{ kN} = 473\text{ kN} \qquad \textbf{Ans.}$$

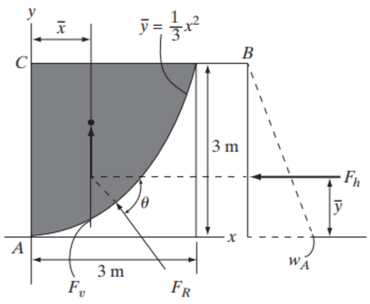


(a)

2-10.

The 5-m-wide overhang is in the form of a parabola. Determine the magnitude and direction of the resultant force on the overhang.





(a)

SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m})(5\ \text{m}) = 147.15\left(10^3\right)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}w_A h_A = \frac{1}{2}\left[147.15\left(10^3\right)\ \text{N/m}\right](3\ \text{m}) = 220.725\left(10^3\right)\ \text{N} = 220.725\ \text{kN}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above surface AB of the wall (shown shaded in Fig. *a*), but acts upward. The volume of this column of water is

$$\forall = \frac{2}{3}ahb = \frac{2}{3}(3\ \text{m})(3\ \text{m})(5\ \text{m}) = 30\ \text{m}^3$$

Thus,

$$F_v = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(30\ \text{m}^3\right) = 294.3\left(10^3\right)\ \text{N} = 294.3\ \text{kN}$$

The magnitude of the resultant force is

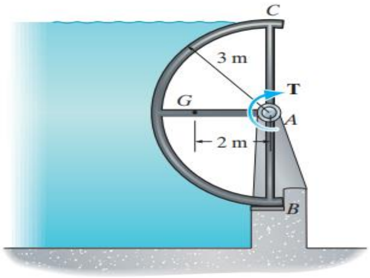
$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(220.725\ \text{kN})^2 + (294.3\ \text{kN})^2} = 367.875\ \text{kN} = 368\ \text{kN} \quad \textbf{Ans.}$$

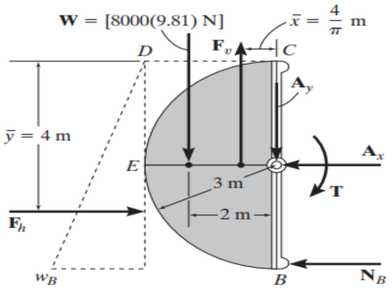
Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{294.3\ \text{kN}}{220.725\ \text{kN}}\right) = 53.13° = 53.1° \quad \textbf{Ans.}$$

2-11.

The semicircular gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque **T** that must be applied at the pin A in order to open the gate. The gate has a mass of 8 Mg with center of mass at G. It is 4 m wide.





(a)

SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width of $b = 4$ m, the intensity of the distributed load at point B is

$$w_B = \rho_w g h_C b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(4\ \text{m}) = 235.44(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w_C h_C = \frac{1}{2}[235.44(10^3)\ \text{N/m}](6\ \text{m}) = 706.32(10^3)\ \text{N}$$

and it acts at

$$\bar{y} = \frac{2}{3}(6\ \text{m}) = 4\ \text{m}$$

The upward force on BE and downward force on CE is equal to the weight of water contained in blocks $BACDEB$ (imaginary) and $CDEC$, respectively. Thus, the net upward force on BEC is equal to the weight of water contained in block $BACEB$ shown shaded in Fig. *a*. Thus,

$$F_v = \rho_w g \forall_{BACEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{2}(3\ \text{m})^2(4\ \text{m})\right]$$

$$= 176.58(10^3)\pi\ \text{N}$$

And it acts at

$$\bar{x} = \frac{4(3\ \text{m})}{3\pi} = \frac{4}{\pi}\ \text{m}$$

When the gate is on the verge of opening, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to the FBD of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad [8000(9.81)\ \text{N}](2\ \text{m}) + [706.32(10^3)\ \text{N}](4\ \text{m} - 3\ \text{m})$$

$$- [176.58(10^3)\pi\ \text{N}]\left(\frac{4}{\pi}\text{m}\right) - T = 0$$

$$T = 156.96(10^3)\ \text{N}\cdot\text{m} = 157\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

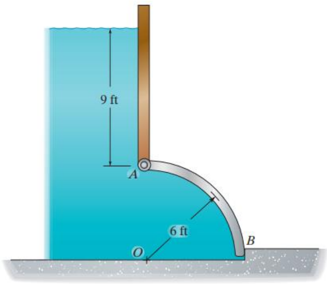
This solution can be simplified if one realizes that the resultant force due to the water pressure on the gate will act perpendicular to the circular surface, thus acting through center A of the semicircular gate and so producing no moment about this point.

$$\circlearrowleft + \Sigma M_A = 0; \quad [8000(9.81)\text{N}](2\text{ m}) - T = 0$$

$$T = 156.96(10^3)\text{ N}\cdot\text{m} = 157\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

2-12.

Determine the horizontal and vertical components of reaction at the hinge A and the horizontal reaction at the smooth surface B *caused* by the water pressure. The plate has a width of 4 ft.



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projecte area of the gate, Fig. *a*. Since the gate has a constant width $b = 4$ ft, the intensities o the horizontal distributed load at A and B are

$$w_A = \gamma_w h_A b = (62.4\ \mathrm{lb/ft^3})(9\ \mathrm{ft})(4\ \mathrm{ft}) = 2246.4\ \mathrm{lb/ft}$$
$$w_B = \gamma_w h_B b = (62.4\ \mathrm{lb/ft^3})(15\ \mathrm{ft})(4\ \mathrm{ft}) = 3744\ \mathrm{lb/ft}$$

Thus,

$$(F_h)_1 = w_A l_{AD} = (2246.4\ \mathrm{lb/ft})(6\ \mathrm{ft}) = 13.4784(10^3)\ \mathrm{lb}$$

$$(F_h)_2 = \frac{1}{2}(w_B - w_A)l_{AD} = \frac{1}{2}(3744\ \mathrm{lb/ft} - 2246.4\ \mathrm{lb/ft})(6\ \mathrm{ft}) = 4.4928(10^3)\ \mathrm{lb}$$

and they act at

$$\tilde{y}_1 = \frac{1}{2}(6\ \mathrm{ft}) = 3\ \mathrm{ft} \quad \tilde{y}_2 = \frac{2}{3}(6\ \mathrm{ft}) = 4\ \mathrm{ft} \quad \tilde{y}_3 = \frac{1}{3}(6\ \mathrm{ft}) = 2\ \mathrm{ft}$$

The vertical force acting on the gate is equal to the weight of the water contained in th imaginary block above the gate (shown shaded in Fig. *a*), but acts upward. For $(F_v)_1$,

$$(F_v)_1 = \gamma_w \forall_{ADEF} = (62.4\ \mathrm{lb/ft^3})[(6\ \mathrm{ft})(9\ \mathrm{ft})(4\ \mathrm{ft})] = 13.4784(10^3)\ \mathrm{lb}$$

And it acts at

$$\bar{x}_1 = \frac{1}{2}(6\ \mathrm{ft}) = 3\ \mathrm{ft}$$

For $(F_v)_2$, we need to refer to the geometry shown in Fig. *b*.
Here,

$$A_{ADB} = A_{ADBO} - A_{ABO} = (6\ \mathrm{ft})(6\ \mathrm{ft}) - \frac{1}{4}[\pi(6\ \mathrm{ft})^2] = (36 - 9\pi)\ \mathrm{ft}^2$$

Then,

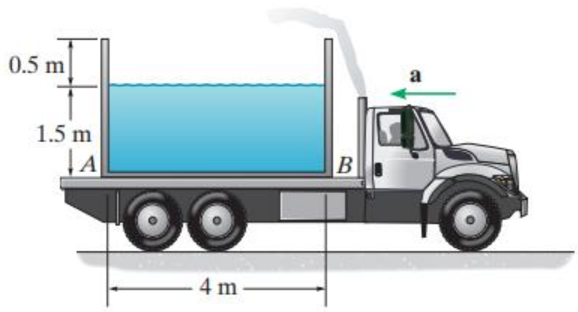
$$(F_v)_2 = \gamma_w \forall_{ADB} = (62.4\ \mathrm{lb/ft^3})[(36 - 9\pi)\ \mathrm{ft}^2](4\ \mathrm{ft}) = 1.9283(10^3)\ \mathrm{lb}$$

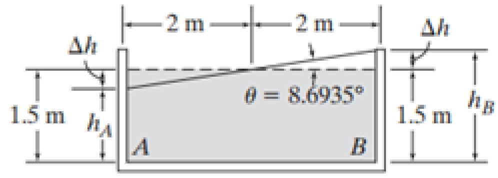
And it acts at

$$\tilde{x}_2 = \frac{(3\ \mathrm{ft})[(6\ \mathrm{ft})(6\ \mathrm{ft})] - \frac{4(6\ \mathrm{ft})}{3\pi}\left[\frac{\pi}{4}(6\ \mathrm{ft})^2\right]}{(36 - 9\pi)\ \mathrm{ft}^2} = 4.6598\ \mathrm{ft}$$

2-13.

The truck carries an open container of water. If it has a constant deceleration $1.5\ \text{m/s}^2$, determine the angle of inclination of the surface of the water and the pressure at the bottom corners A and B.





(a)

SOLUTION

The free surface of the water in the decelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_c}{g} = \frac{1.5\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 8.6935° = 8.69° \qquad \textbf{Ans.}$$

From the geometry in Fig. *a*,

$$\Delta h = (2\ \text{m})\tan 8.6935° = 0.3058\ \text{m}$$

Since $\Delta h < 0.5\ \text{m}$, the water will not spill. Thus,
$h_A = 1.5\ \text{m} - 0.3058\ \text{m} = 1.1942\ \text{m}$ and $h_B = 1.5\ \text{m} + 0.3058\ \text{m} = 1.8058\ \text{m}$. Then

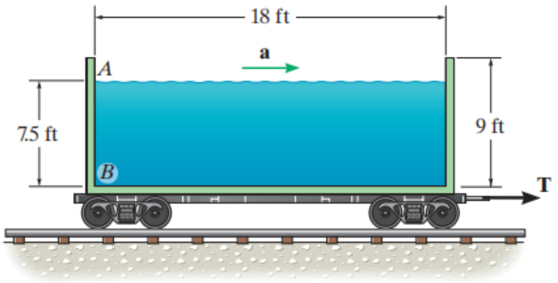
$$p_A = \rho_w g h_A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.1942\ \text{m})$$
$$= 11.715(10^3)\ \text{N/m}^2 = 11.7\ \text{kPa} \qquad \textbf{Ans.}$$

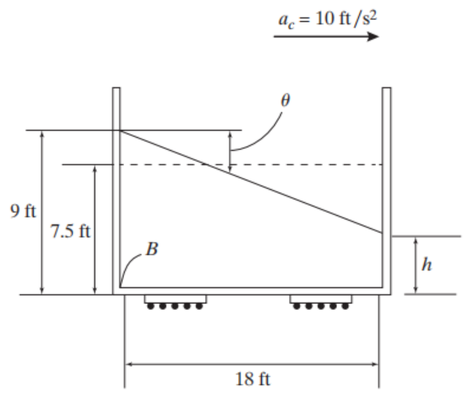
And

$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.8058\ \text{m})$$
$$= 17.715(10^3)\ \text{N/m}^2 = 17.7\ \text{kPa} \qquad \textbf{Ans.}$$

2-14.

The open rail car is 6 ft wide and filled with water to the level shown. Determine the pressure that acts at point B both when the car is at rest, and when the car has a constant acceleration of 10 ft/s^2. How much water spills out of the car?





(a)

SOLUTION

When the car is at rest, the water is at the level shown by the dashed line shown in Fig. *a*.

At rest: $p_B = \gamma_w h_B = (62.4 \text{ lb/ft}^3)(7.5 \text{ ft}) = 468 \text{ lb/ft}^2$ **Ans.**

When the car accelerates, the angle θ the water level makes with the horizontal can be determined.

$$\tan\theta = \frac{a_c}{g} = \frac{10 \text{ ft/s}^2}{32.2 \text{ ft/s}^2}; \qquad \theta = 17.25°$$

Assuming that the water will spill out, then the water level when the car accelerates is indicated by the solid line shown in Fig. *a*. Thus,

$$h = 9 \text{ ft} - 18 \text{ ft} \tan 17.25° = 3.4099 \text{ ft}$$

The original volume of water is

$$\forall = (7.5 \text{ ft})(18 \text{ ft})(6 \text{ ft}) = 810 \text{ ft}^3$$

The volume of water after the car accelerates is

$$\forall' = \frac{1}{2}(9 \text{ ft} + 3.4099 \text{ ft})(18 \text{ ft})(6 \text{ ft}) = 670.14 \text{ ft}^3 < 810 \text{ ft}^3 \qquad \textbf{(OK!)}$$

Thus, the amount of water spilled is

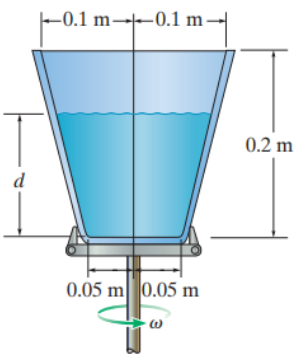
$$\Delta\forall = \forall - \forall' = 810 \text{ ft}^3 - 670.14 \text{ ft}^3 = 139.86 \text{ ft}^3 = 140 \text{ ft}^3 \qquad \textbf{Ans.}$$

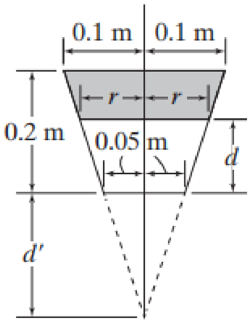
The pressure at B when the car accelerates is

With acceleration: $p_B = \gamma_w h_B = (62.4 \text{ lb/ft}^3)(9 \text{ ft}) = 561.6 \text{ lb/ft}^2 = 562 \text{ lb/ft}^2$ **Ans.**

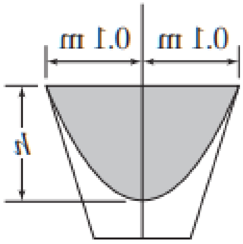
2-15.

Determine the maximum height d the glass can be filled with water so that no water spills out when the glass is rotating at 15 rad/s.





(a)



(b)

SOLUTION

From the geometry shown in Fig. *a*,

$$\frac{d'}{d' + 0.2\text{ m}} = \frac{0.05\text{ m}}{0.1\text{ m}}; \quad d' = 0.2\text{ m}$$

Then

$$\frac{r}{0.05\text{ m}} = \frac{d + 0.2\text{ m}}{0.2\text{ m}}; \qquad r = 0.25(d + 0.2)$$

Thus, the volume of the empty space in the container shown shaded in Fig. *a* is

$$\begin{aligned} V_{es} &= \frac{1}{3}\pi(0.1\text{ m})^2(0.4\text{ m}) - \frac{1}{3}\pi[0.25(d + 0.2)]^2(d + 0.2) \\ &= \frac{1}{3}\pi[0.004 - 0.0625(d + 0.2)^3]\text{m}^3 \end{aligned}$$

For the condition that the water is about to spill, the parabolic profile of the free water surface is shown in Fig. *b*.

$$h = \left(\frac{\omega^2}{2g}\right)r^2$$

$$\begin{aligned} h &= \left[\frac{(15\text{ rad/s})^2}{2(9.81\text{ m/s}^2)}\right](0.1\text{ m})^2 \\ &= 0.1147\text{ m} < 0.2\text{ m} \end{aligned} \qquad \textbf{(O.K!)}$$

Since the empty space in the glass must remain the same, the volume of the paraboloid shown shaded in Fig. *b* must be equal to this volume. Here, the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height.

$$V_{\text{parab}} = V_{\text{es}}$$

$$\frac{1}{2}[\pi(0.1\text{ m})^2](0.1147\text{ m}) = \frac{1}{3}\pi[0.004 - 0.0625(d + 0.2)^3]$$

$$d = 0.1316\text{ m} = 0.132\text{ m} \qquad \textbf{Ans.}$$

Chapter 3 Differential Relations for a Fluid Particle

3-1.

A flow field for gasoline is defined by $u = \left(\frac{8}{y}\right)$ m/s, and $v = (2x)$ m/s where x and y are in meters. Determine the equation of the streamline that passes through point (2 m, 4 m). Draw this streamline.

SOLUTION

Using the definition of the slope of streamline and initial condition $y = 4$ m at $x = 2$ m,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{2x}{8/y}$$

$$\int_{4\text{ m}}^{y} \frac{dy}{y} = \frac{1}{4}\int_{2\text{ m}}^{x} x\,dx$$

$$\ln y\Big|_{4\text{ m}}^{y} = \frac{1}{8}x^2\Big|_{2\text{ m}}^{x}$$

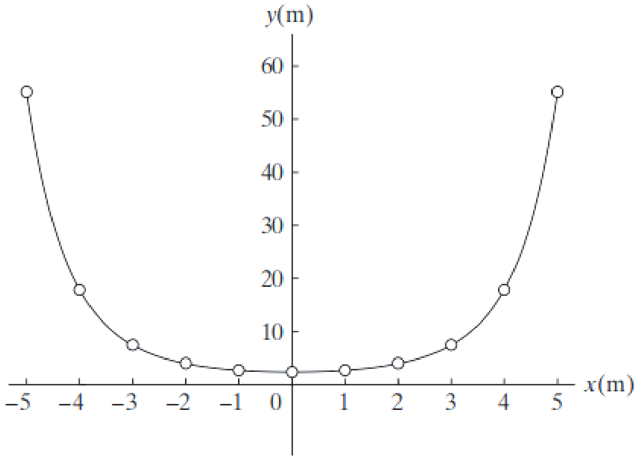
$$\ln\frac{y}{4} = \frac{1}{8}(x^2 - 4)$$

$$y = 4e^{\frac{1}{8}(x^2-4)}$$ **Ans.**

The values of x and the corresponding values of y are tabulated below:

x(m)	0	± 1	± 2	± 3	± 4	4
y(m)	2.43	2.75	4	7.47	17.9	55.2

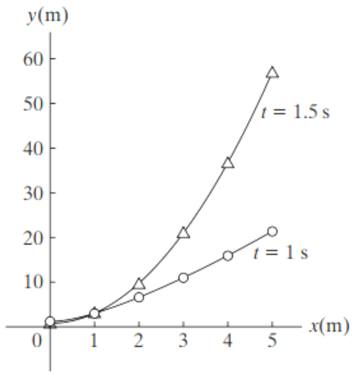
The plot of y vs x is shown in Fig. *a*.



(a)

3-2

A fluid has velocity components of $u = (3x^2 + 1)$ m/s and $v = (4txy)$ m/s, where x and y are in meters and t is in seconds. Determine the streamlines that pass through point (1 m, 3 m) at times $t = 1$ s and $t = 1.5$ s. Plot these streamlines for $0 \le x \le 5$ m.



(a)

SOLUTION

Since the velocity components are functions of time and position, the flow can be classified as unsteady nonuniform flow. The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{4txy}{3x^2+1} = 4ty\left(\frac{x}{3x^2+1}\right)$$

$$\int_{3\,\text{m}}^{y} \frac{dy}{y} = 4t\int_{1\,\text{m}}^{x}\left(\frac{x}{3x^2+1}\right)dx$$

$$\ln y\Big|_{3\,\text{m}}^{y} = \frac{2t}{3}\ln(3x^2+1)\Big|_{1\,\text{m}}^{x}$$

$$\ln\frac{y}{3} = \frac{2t}{3}\ln\left(\frac{3x^2+1}{4}\right)$$

$$\ln\frac{y}{3} = \ln\left[\left(\frac{3x^2+1}{4}\right)^{2t/3}\right]$$

$$y = \left[3\left(\frac{3x^2+1}{4}\right)^{2t/3}\right]\text{m}$$

For $t = 1$ s,

$$y = \left[3\left(\frac{3x^2+1}{4}\right)^{2/3}\right]\text{m} \qquad \textbf{Ans.}$$

For $t = 1.5$ s,

$$y = \left[\frac{3}{4}(3x^2+1)\right]\text{m} \qquad \textbf{Ans.}$$

The values of x and the corresponding values of y are tabulated below.
For $t = 1$ s,

x(m)	0	1	2	3	4	5
y(m)	1.19	3	6.58	11.0	15.9	21.4

For $t = 1.5$ s,

x(m)	0	1	2	3	4	5
y(m)	0.75	3	9.75	21	36.75	57

The plot of these streamlines are shown in Fig. *a*.

3-3.

Air flows uniformly through the center of a horizontal duct with a velocity of $V = \left(\frac{1}{4}t^3 + 3\right)$ m/s, where t is in seconds. Determine the acceleration of the flow when $t = 3$ s.

SOLUTION

Since the flow is along the horizontal (x axis), $v = w = 0$ and $u = V$. Also, the velocity is a function of time t only. Therefore, the convective acceleration is zero, so that $V\dfrac{\partial V}{\partial x} = 0$.

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

$$= \frac{3}{4}t^2 + 0$$

$$= \left(\frac{3}{4}t^2\right) \text{m/s}^2$$

When $t = 3$ s,

$$a = \frac{3}{4}(3^2) = 6.75 \text{ m/s}^2$$ **Ans.**

Note: The flow is unsteady since its velocity is a function of time.

3-4.

A fluid has velocity components of $u = \left(\frac{1}{16}x^2yt\right)$ ft/s and $v = (6x - t)$ ft/s where x and y are in feet and t is in seconds. Determine the magnitude of acceleration of a particle passing through the point (2 ft, 1 ft) if it arrives when $t = 2$ s.

SOLUTION

For two-dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Write the scalar components of this equation along x and y axes.

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= \frac{1}{16}x^2y + \frac{1}{16}x^2yt\left(\frac{1}{8}xyt\right) + (6x - t)\left(\frac{1}{16}x^2t\right)$$

$$= \left[\frac{1}{16}x^2\left(y + \frac{1}{8}y^2t^2 + 6xt - t^2\right)\right]\ \text{ft/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= -1 + \left(\frac{1}{16}x^2yt\right)(6) + (6x - t)(0)$$

$$= \left(\frac{3}{8}x^2yt - 1\right)\ \text{ft/s}^2$$

When $t = 2$ s, $x = 2$ ft and $y = 1$ ft.

$$a_x = \frac{1}{16}(2^2)\left[1 + \frac{1}{8}(2)(1^2)(2^2) + 6(2)(2) - 2^2\right] = 5.5\ \text{ft/s}^2$$

$$a_y = \frac{3}{8}(2^2)(1)(2) - 1 = 2\ \text{ft/s}^2$$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(5.5\ \text{ft/s}^2)^2 + (2\ \text{ft/s}^2)^2} = 5.85\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

3-5.

A fluid flow is defined by $u = (6x^2 - 3y^2)$ m/s and $v = (4xy + y)$ m/s, where x and y are in meters. Determine the magnitudes of the velocity and acceleration of a particle at point (2 m, 2 m).

SOLUTION

Since the velocity components are a function of position only, the flow can be classified as steady nonuniform flow. The x and y velocity components of the particles at $x = 2$ m and $y = 2$ m are

$$u = 6(2^2) - 3(2^2) = 12 \text{ m/s}$$

$$v = 4(2)(2) + 2 = 18 \text{ m/s}$$

The magnitude of the particle's velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(12 \text{ m/s})^2 + (18 \text{ m/s})^2} = 21.63 \text{ m/s} = 21.6 \text{ m/s} \qquad \textbf{Ans.}$$

The x and y components of the particle's acceleration, with $w = 0$, are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (6x^2 - 3y^2)(12x) + (4xy + y)(-6y)$$

$$= (72x^3 - 60xy^2 - 6y^2) \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (6x^2 - 3y^2)(4y) + (4xy + y)(4x + 1)$$

$$= (40x^2y - 12y^3 + 8xy + y) \text{ m/s}^2$$

The magnitude of the particle's acceleration is

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{(72 \text{ m/s}^2) + (258 \text{ m/s}^2)} = 267.86 \text{ m/s}^2 = 268 \text{ m/s}^2 \qquad \textbf{Ans.}$$

At $x = 2$ m and $y = 2$ m,

$$a_x = 72(2^3) - 60(2)(2^2) - 6(2^2) = 72 \text{ m/s}^2$$

$$a_y = 40(2^2)(2) - 12(2^3) + 8(2)(2) + 2 = 258 \text{ m/s}^2$$

3-6.

A fluid flow is defined by $u = (4xy)$ ft/s and $v = (3y)$ ft/s, where x and y are in feet. Determine the equation of the streamline passing through point (1 ft, 2 ft). Also find the acceleration of a particle located at this point. Is the flow steady or unsteady?

Since the velocity components are the function of position but not the time, **the flow is steady** (**Ans.**), but nonuniform. Using the definition of the slope of the streamline,

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{3y}{4xy} = \frac{3}{4x}$$

$$\int_{2\text{ ft}}^{y} dy = \frac{3}{4}\int_{1\text{ ft}}^{x} \frac{dx}{x}$$

$$y\Big|_{2\text{ ft}}^{y} = \frac{3}{4}\ln x\Big|_{1\text{ ft}}^{x}$$

$$y - 2 = \frac{3}{4}\ln x$$

$$y = \left(\frac{3}{4}\ln x + 2\right)\text{ ft} \qquad \textbf{Ans.}$$

For two dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Write the scalar components of this equation along x and y axes.

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 0 + (4xy)(4y) + 3y(4x)$$

$$= \left[4xy(4y + 3)\right]\text{ ft/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (4xy)(0) + 3y(3)$$

$$= (9y)\text{ ft/s}^2$$

At $x = 1$ ft, $y = 2$ ft,

$$a_x = 4(1)(2)\left[4(2) + 3\right] = 88\text{ ft/s}^2 \rightarrow \qquad a_y = 9(2) = 18\text{ ft/s}^2 \uparrow$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} = \sqrt{(88\text{ ft/s}^2)^2 + (18\text{ ft/s}^2)^2} = 89.8\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{18\text{ ft/s}^2}{88\text{ ft/s}^2}\right) = 11.6^\circ \qquad \textbf{Ans.}$$

3-7.

A fluid flow is defined by $u = (8t^2)$ m/s and $v = (7y + 3x)$ m/s, where x and y are in meters and t is in seconds. Determine the velocity and acceleration of a particle passing through point (1 m, 1 m) if it arrives when $t = 2$ s.

SOLUTION

Since the velocity components are functions of time and position, the flow can be classified as unsteady nonuniform flow. When $t = 2$ s, $x = 1$ m and $y = 1$ m.

$$u = 8(2^2) = 32 \text{ m/s}$$

$$v = 7(1) + 3(1) = 10 \text{ m/s}$$

The magnitude of the velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(32 \text{ m/s})^2 + (10 \text{ m/s})^2} = 33.5 \text{ m/s} \qquad \textbf{Ans.}$$

Its direction is

$$\theta_v = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(\frac{10 \text{ m/s}}{32 \text{ m/s}}\right) = 17.4° \measuredangle \theta_v \qquad \textbf{Ans.}$$

For two-dimensional flow, the Eulerian description gives

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= 16t + 8t^2(0) + (7y + 3x)(0)$$

$$= (16t) \text{ m/s}^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + (8t^2)(3) + (7y + 3x)(7)$$

$$= \left[24t^2 + 7(7y + 3x)\right] \text{ m/s}^2$$

When $t = 2$ s, $x = 1$ m and $y = 1$ m.

$$a_x = 16(2) = 32 \text{ m/s}^2$$

$$a_y = 24(2^2) + 7[7(1) + 3(1)] = 166 \text{ m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(32 \text{ m/s}^2)^2 + (166 \text{ m/s}^2)^2} = 169 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Its direction is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{166 \text{ m/s}^2}{32 \text{ m/s}^2}\right) = 79.1° \measuredangle \theta_a \qquad \textbf{Ans.}$$

3-8.

Fluid particles have velocity components of $u = (2x)$ m/s and $v = (4y)$ m/s, where x and y are in meters. Determine the acceleration of a particle located at point (2 m, 1 m). Determine the equation of the streamline passing through this point.

SOLUTION

Since the velocity components are independent of time, but a function of position, the flow can be classified as steady nonuniform flow. For two-dimensional flow, $(w = 0)$, the Eulerian description is

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

Writing the scalar components of this equation along the x and y axes,

$$\begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ &= 0 + 2x(2) + 4y(0) \\ &= (4x)\ \mathrm{m/s^2} \\ a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ &= 0 + 2x(0) + 4y(4) \\ &= (16y)\ \mathrm{m/s^2} \end{aligned}$$

At point $x = 2$ m and $y = 1$ m,

$$a_x = 4(2) = 8\ \mathrm{m/s^2} \rightarrow$$
$$a_y = 16(1) = 16\ \mathrm{m/s^2} \uparrow$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(8\ \mathrm{m/s^2})^2 + (16\ \mathrm{m/s^2})^2} = 17.89\ \mathrm{m/s^2} = 17.9\ \mathrm{m/s^2} \quad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{16\text{ m/s}^2}{8\text{ m/s}^2}\right) = 63.43° = 63.4° \measuredangle \qquad \textbf{Ans.}$$

The slope of the streamline is

$$\frac{dy}{dx} = \frac{v}{u}; \qquad \frac{dy}{dx} = \frac{4y}{2x} = \frac{2y}{x}$$

$$\int_{1\text{ m}}^{y} \frac{dy}{y} = 2\int_{2\text{ m}}^{x} \frac{dx}{x}$$

$$\ln y\Big|_{1\text{ m}}^{y} = 2(\ln x)\Big|_{2\text{ m}}^{x}$$

$$\ln y = 2\ln\frac{x}{2}$$

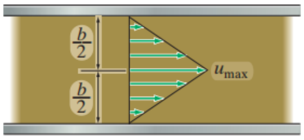
$$\ln y = \ln\left[\left(\frac{x}{2}\right)^2\right]$$

$$y = \left(\frac{x}{2}\right)^2$$

$$y = \frac{1}{4}x^2 \qquad \textbf{Ans.}$$

3-9.

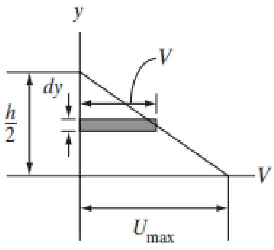
A fluid flowing between two plates has a velocit profile that is assumed to be linear as shown. Determine the average velocity and volumetric discharge in terms of u_{max}. The plates have a width of w.



SOLUTION

The velocity profile in Fig. *a* can be expressed as

$$\frac{v - 0}{y - \frac{h}{2}} = \frac{U_{max} - 0}{0 - \frac{h}{2}}; \qquad v = U_{max}\left(1 - \frac{2}{h}y\right)$$



(a)

The differential rectangular element of the thickness dy on the cross section will be considered. Thus, $dA = wdy$.

$$Q = \int_A v = dA$$

$$= 2\int_0^{\frac{h}{2}} \left[U_{max}\left(1 - \frac{2}{h}y\right)\right](wdy)$$

$$= 2wU_{max}\int_0^{\frac{h}{2}}\left(1 - \frac{2}{h}y\right)dy$$

$$= 2wU_{max}\left(y - \frac{y^2}{h}\right)\Bigg|_0^{\frac{h}{2}}$$

$$= \frac{wU_{max}h}{2} \qquad \textbf{Ans.}$$

Also,

$$Q = \int_A v \cdot d\mathbf{A} = \text{volume under velocity diagram}$$

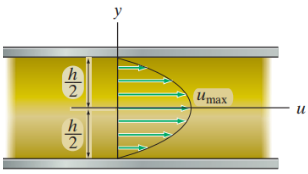
$$= \frac{1}{2}(h)(w)(U_{max}) = \frac{wU_{max}h}{2} \qquad \textbf{Ans.}$$

Therefore,

$$V = \frac{Q}{A} = \frac{wU_{max}h}{2(w)(h)} = \frac{U_{max}}{2} \qquad \textbf{Ans.}$$

3-10.

A fluid flowing between two plates has a velocity profile that is assumed to be parabolic, where $u = \frac{4u_{\max}}{h^2}(hy - y^2)$. Determine the average velocity and volumetric discharge in terms of $u_{\max}$. The plates have a width of w.



SOLUTION

The differential rectangular element of thickness dy shown shaded in Fig. *a* having base area of $dA = wdy$ will be considered. Thus,

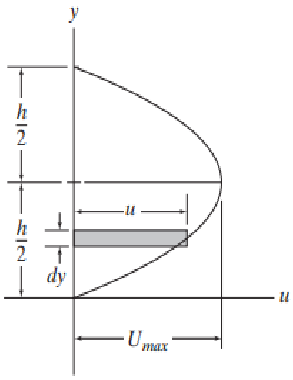
$$Q = \int_A u\,dA$$

$$= \int_0^h \frac{4u_{\max}}{h^2}(hy - y^2)(wdy)$$

$$= \frac{4wu_{\max}}{h^2}\int_0^h (hy - y^2)dy$$

$$= \frac{4wu_{\max}}{h^2}\left(\frac{hy^2}{2} - \frac{y^3}{3}\right)\Bigg|_0^h$$

$$= \frac{2}{3}whu_{\max} \qquad \textbf{Ans.}$$



(a)

Also, since the velocity profile is a parabola,

$$Q = \int_A \mathbf{V}\cdot d\mathbf{A} = \text{volume under the velocity diagram}$$

$$= \left[\frac{2}{3}(h)(u_{\max})\right]w$$

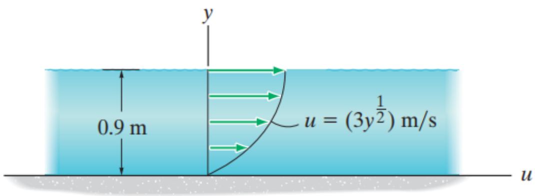
$$= \frac{2}{3}whu_{\max} \qquad \textbf{Ans.}$$

Therefore

$$V = \frac{Q}{A} = \frac{\frac{2}{3}whu_{\max}}{wh} = \frac{2}{3}u_{\max} \qquad \textbf{Ans.}$$

3-11.

The liquid in the rectangular channel has a velocity profile defined by $u = (3y^{1/2})$ m/s, where y is in meters. Determine the volumetric discharge if the width of the channel is 2 m.



SOLUTION

The differential rectangular element of thickness dy shown shaded in Fig. a having base area of $dA = (2\text{ m})dy$ will be considered. Thus,

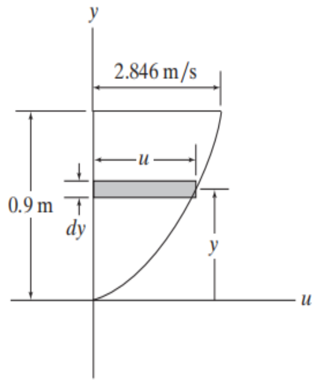
$$Q = \int_A v\,dA$$

$$= \int_0^{0.9\text{ m}} (3y^{1/2})(2dy)$$

$$= 6\int_0^{0.9\text{ m}} y^{1/2}dy$$

$$= 6\left(\frac{2}{3}y^{3/2}\right)\Bigg|_0^{0.9\text{ m}}$$

$$= 3.415\text{ m}^3/\text{s} = 3.42\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$



(a)

Also, when $y = 0.9$ m, $u = 3(0.9^{1/2}) = 2.846$ m/s. Since the velocity profile is a half-parabola,

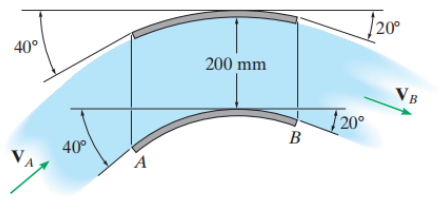
$$Q = \int \mathbf{V}\cdot d\mathbf{A} = \text{The volume under the velocity diagram}$$

$$= \left[\frac{2}{3}(0.9\text{ m})(2.846\text{ m/s})\right](2\text{ m})$$

$$= 3.415\text{ m}^3/\text{s} = 3.42\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

3-12.

Air flows through the gap between the vanes at $0.75\ \mathrm{m^3/s}$. Determine the velocity of the air passing through the inlet A and the outlet B. The vanes have a width of 400 mm and the vertical distance between them is 200 mm.



SOLUTION

The discharge can be calculated using

$$Q = \int_{cs} \mathbf{V}_a \cdot d\mathbf{A}$$

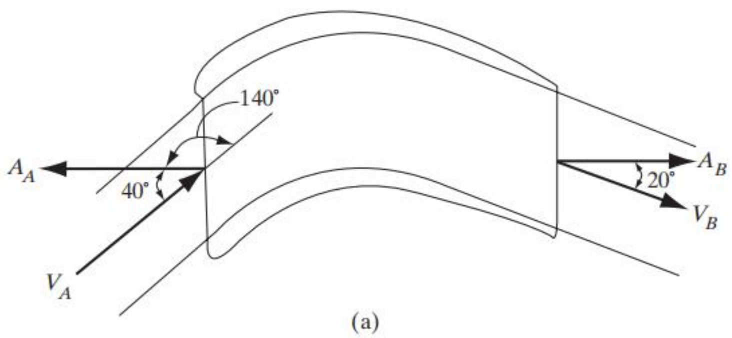
Here, the average velocities will be used. Referring to Fig. *a*,

$$Q_A = \mathbf{V}_A \cdot \mathbf{A}_A; \qquad -0.75\ \mathrm{m^3/s} = (V_A \cos 140°)\left[(0.2\ \mathrm{m})(0.4\ \mathrm{m})\right]$$

$$V_A = 12.2\ \mathrm{m/s} \qquad \textbf{Ans.}$$

$$Q_B = \mathbf{V}_B \cdot \mathbf{A}_B; \qquad 0.75\ \mathrm{m^3/s} = (V_B \cos 20°)\left[(0.2\ \mathrm{m})(0.4\ \mathrm{m})\right]$$

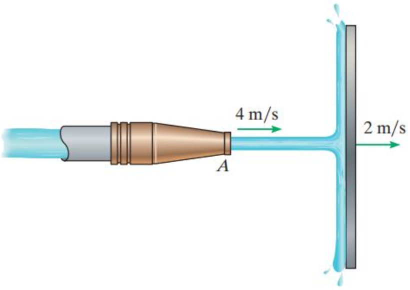
$$V_B = 9.98\ \mathrm{m/s} \qquad \textbf{Ans.}$$



(a)

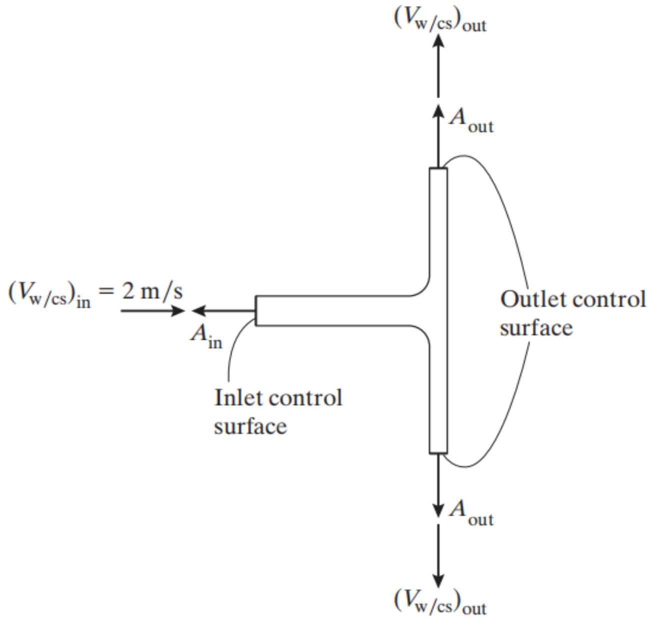
3-13.

The flat plate is moving to the right at 2 m/s. Water is ejected from the nozzle at A at an average velocity of 4 m/s. Outline a moving control volume that contains the water on the plate. Indicate the open control surfaces, and show the positive direction of their areas through which flow occurs. Also, indicate the directions of the relative velocities through the control surfaces and determine the magnitude of the relative velocity through the inlet control surface. Identify the local and convective changes that occur. Assume water to be incompressible. For the analysis, why is it best to consider the control volume as moving?



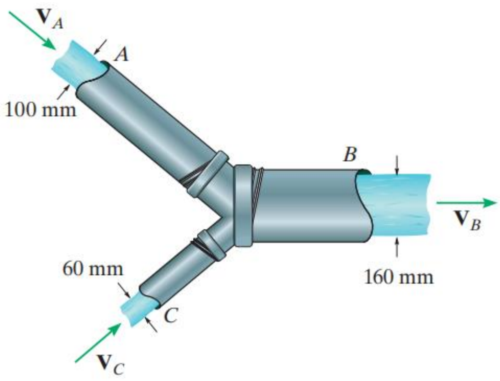
SOLUTION

If the control volume is considered moving with the plate, then the flow can be considered steady as measured relative to the control volume. No local changes occur. Also, the water flows in and out through the open (inlet and outlet) control surfaces. This causes convective changes.



3-14.

Water flows through the pipe at A at 60 kg/s, and then out of B with a velocity of 4 m/s. Determine the average velocity at which it flows in through C.



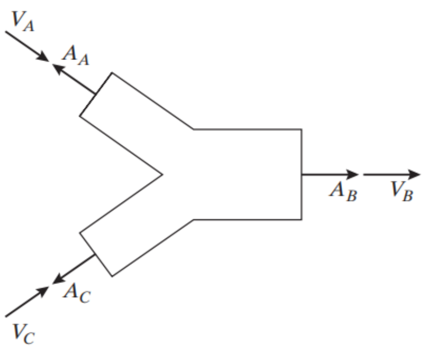
SOLUTION

The fixed control volume considered is shown in Fig. a. Since the flow is steady, there is no change in volume, and therefore no local changes occur within this control volume. For the flow at A,

$$\dot{m}_A = \rho V_A A_A$$
$$60 \text{ kg/s} = (1000 \text{ kg/m}^3)(V_A A_A)$$
$$V_A A_A = 0.06 \text{ m}^3/\text{s}$$

Since the fluid is water, which has a constant density, then the continuity equation can be simplified as

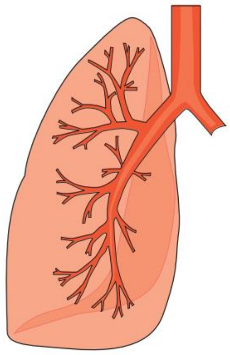
$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$
$$0 + V_B A_B - V_A A_A - V_C A_C = 0$$
$$(4 \text{ m/s})[\pi(0.08 \text{ m})^2] - 0.06 \text{ m}^3/\text{s} - V_C[\pi(0.03 \text{ m})^2] = 0$$
$$V_C = 7.224 \text{ m/s} = 7.22 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

3-15.

With every breath, air enters the trachea, its flow split equally into two main bronchi, and then passes through about 150 000 bronchial tubes before entering the alveoli. If the air flow into the 18-mm-diameter trachea is 12 liter/min., determine the velocity of the air in the trachea and the main bronchi, which have a diameter of 12 mm. *Note:* The diameter of the alveoli is about 250 μm, and because there are so many of them, the flow is reduced to practically zero, so gaseous exchange is carried out by diffusion.



SOLUTION

The fixed control volume considered is the air contained within the trachea and two main bronchi shown in Fig. *a*. Since the volume of this control volume does not change with time, no local changes occur within this control volume. Here, the density of the air is considered constant, and so the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv} \rho d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_t A_t + 2V_b A_b = 0 \qquad \textbf{(1)}$$

Here, $Q = \left(12\ \frac{\cancel{\text{liter}}}{\cancel{\text{min}}}\right)\left(\frac{1\ \text{m}^3}{1000\ \cancel{\text{liter}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right) = 0.2(10^{-3})\ \text{m}^3/\text{s}$. The volumetric flow rate gives

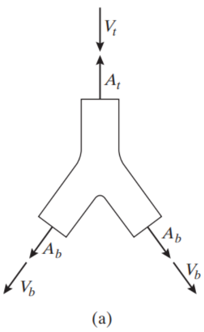
$$Q = V_t A_t; \quad 0.2(10^{-3})\ \text{m}^3/\text{s} = V_t[\pi(0.009\ \text{m})^2]$$

$$V_t = 0.7860\ \text{m/s} = 0.786\ \text{m/s} \qquad \textbf{Ans.}$$

Then Eq. (1) gives

$$-0.2(10^{-3})\ \text{m}^3/\text{s} + 2V_b[\pi(0.006\ \text{m})^2]$$

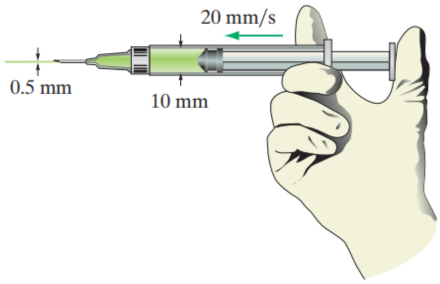
$$V_b = 0.8842\ \text{m/s} = 0.884\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

3-16.

The cylindrical syringe is actuated by applying a force on the plunger. If this causes the plunger to move forward at 20 mm/s, determine the velocity of the fluid passing out of the needle.



SOLUTION

The deformable control volume considered is shown in Fig. *a*. If the volume of the control volume is initially $\forall_0$ then at any instant its volume is

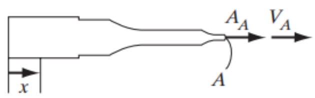
$$\forall = \forall_0 - \frac{\pi}{4}(0.01\text{ m})^2 x = \left[\forall_0 - 25(10^{-6})\pi x\right]\text{ m}^3$$

With the fluid assumed to be incompressible, ρ is constant. Since $\mathbf{V}_A$ and $\mathbf{A}_A$ are in the same sense, $Q_A = V_A A_A = V_A\left[\frac{\pi}{4}(0.0005\text{ m})^2\right] = \left[62.5(10^{-9})\pi V_A\right]\text{ m}^3/\text{s}$.

Then, the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\mathbf{V}\cdot d\mathbf{A} = 0$$

$$\rho\left[\frac{\partial}{\partial t}(\forall) + V_A A_A\right] = 0$$



(a)

$$\frac{\partial}{\partial t}\left[\forall_0 - 25(10^{-6})\pi x\right] + 62.5(10^{-9})\pi V_A = 0$$

$$-25(10^{-6})\pi\frac{dx}{dt} + 62.5(10^{-9})\pi V_A = 0$$

However,

$$\frac{dx}{dt} = 20\text{ mm/s} = 0.02\text{ m/s}$$

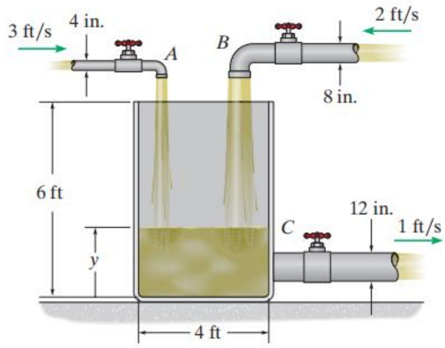
Then

$$\left[-25(10^{-6})\pi\right](0.02) + 62.5(10^{-9})\pi V_A = 0$$

$$V_A = 8.00\text{ m/s}$$ **Ans.**

3-17.

Kerosene flows into the rectangular tank through pipes A and B, with velocities of 3 ft/s and 2 ft/s, respectively. It exits at C with a velocity of 1 ft/s. Determine the rate at which the surface of the kerosene is rising. The base of the tank is 6 ft by 4 ft. Ignore the effect of gravity on the falling kerosene.



SOLUTION

The control volume is the volume of the kerosene in the tank including the two downflows. Thus its volume changes with time.

$$\frac{\partial}{\partial t}\int_{cv} \rho_{ke}\, d\forall + \int_{cs} \rho_{ke}\mathbf{V}\cdot d\mathbf{A} = 0$$

Since ρ_{ke} is constant (incompressible), it can be factored out of the integral.

$$\rho_{ke}\frac{\partial \forall}{\partial t} + \rho_{ke}\int_{cs} \mathbf{V}\cdot d\mathbf{A} = 0$$

Here, we will use the average velocities.

$$\frac{d\forall}{dt} - V_A A_A - V_B A_B + V_C A_C = 0$$

$$\frac{d\forall}{dt} - (3\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{4}{12}\text{ft}\right)^2\right] - (2\text{ ft/s})\left[\frac{\pi}{4}\left(\frac{8}{12}\text{ft}\right)^2\right] + (1\text{ ft/s})\left[\frac{\pi}{4}(1\text{ ft})^2\right] = 0$$

$$\frac{d\forall}{dt} = \frac{\pi}{18}\text{ft}^3/\text{s} \qquad \textbf{(1)}$$

The volume of the control volume at a particular instant is

$$\forall = (6\text{ ft})(4\text{ ft})y + \frac{\pi}{4}\left(\frac{1}{3}\text{ft}\right)^2(6-y) + \frac{\pi}{4}\left(\frac{2}{3}\text{ft}\right)^2(6-y) = \left[\left(24 - \frac{5\pi}{36}\right)y + \frac{5\pi}{6}\right]\text{ft}^3$$

Thus

$$\frac{\partial \forall}{\partial t} = \left(24 - \frac{5\pi}{36}\right)\frac{\partial y}{\partial t}$$

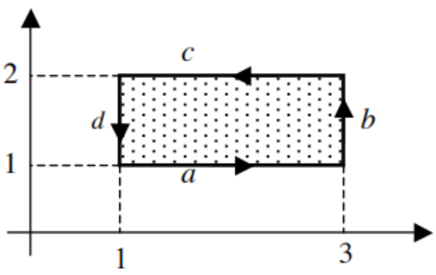
Substituting this result into Eq. (1),

$$\left(24 - \frac{5\pi}{6}\right)\frac{\partial y}{\partial t} = \frac{\pi}{18}$$

$$\frac{dy}{dt} = 0.00816\text{ ft/s} \qquad \textbf{Ans.}$$

3-18.

For the velocity distribution $u = -B\,y$, $v = +B\,x$, $w = 0$, evaluate the circulation Γ about the rectangular closed curve defined by $(x, y) = (1,1)$, $(3,1)$, $(3,2)$, and $(1,2)$. Interpret your result, especially vis-à-vis the velocity potential.



SOLUTION

Given that $\Gamma = \int \mathbf{V}\cdot d\mathbf{s}$ around the curve, divide the rectangle into (a, b, c, d) pieces as shown.

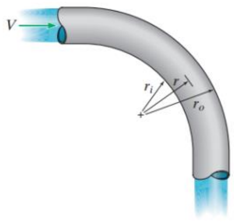
$$\Gamma = \int_a u\,ds + \int_b v\,ds + \int_c u\,ds + \int_d v\,ds = (-B)(2) + (3B)(1) + (-2B)(2) + (-B)(1) = +\mathbf{4B} \quad Ans.$$

The flow is *rotational*. Check $|\mathrm{curl}\,\mathbf{V}| = 2B = \text{constant}$, so $\Gamma = (2B)A_{region} = (2B)(2) = 4B$.

Chapter 4 Integral Relations for a Control Volume

4-1.

An ideal fluid having a density ρ flows with a velocity V through the *horizontal* pipe bend. Plot the pressure variation within the fluid as a function of the radius r, where $r_i \le r \le r_o$ and $r_o = 2r_i$. For the calculation, assume the velocity is constant over the cross section.



SOLUTION

Since the fluid is inviscid (ideal fluid) and the flow is steady (constant V) and along the circular bend, Euler's differential equation in the n-direction can be applied

$$-\frac{dp}{dn} - \rho g\frac{dz}{dn} = \frac{\rho V^2}{R}$$

Since the pipe lies in the horizontal p ane, the elevation term (second term on the left) can be excluded. Also the n axis and r are opposite in sense thus, $dn = -dr$. with $R = r$

$$\frac{dp}{dr} = \frac{\rho V^2}{r}$$

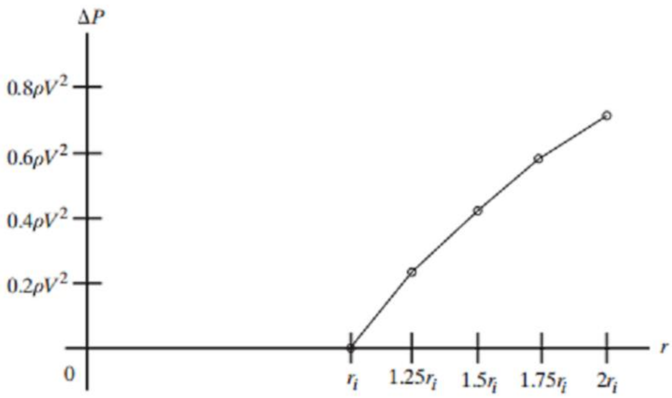
$$\int dp = \rho V^2 \int_{r_i}^{r} \frac{dr}{r}$$

$$\Delta p = \rho V^2 \ln\frac{r}{r_i}$$

The tabulation for $r_i \le r \le 2r_i$ is calculated below.

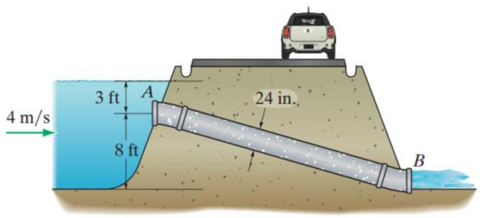
r	r_i	$1.25\ r_i$	$1.50\ r_i$	$1.75\ r_i$	$2\ r_i$
Δp	0	$0.223\ \rho V^2$	$0.405\ \rho V^2$	$0.560\ \rho V^2$	$0.693\ \rho V^2$

The plot of this relation is shown in Fig. *a* **Ans.**



4-2.

The water in an open channel drainage canal flows with a velocity of $V_A = 4$ m/s into the drainpipe that crosses a highway embankment. Determine the volumetric discharge through the pipe. Neglect any head losses.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Since point B is exposed to the atmosphere, $p_B = p_{\text{atm}} = 0$. Here, $V_A = 4$ ft/s with reference to the datum set through point B, $z_A = 8\text{ ft} + 3\text{ ft} = 11\text{ ft}$ and $z_B = 0$. The pressure head at A is (3 ft). Applying the Bernoulli's equation between points A and B,

$$\frac{p_A}{\rho_w} + \frac{{V_A}^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{{V_B}^2}{2} + gz_B$$

$$(3\text{ ft})(32.2\text{ ft/s}^2) + \frac{(4\text{ ft/s})^2}{2} + (32.2\text{ ft/s}^2)(11\text{ ft}) = 0 + \frac{V_B^2}{2} + 0$$

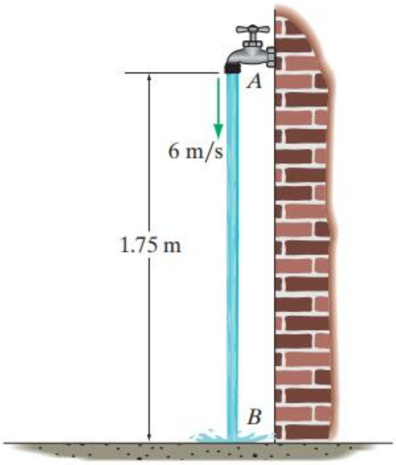
$$V_B = 30.29\text{ ft/s}$$

Thus, the volumetric flow rate is given by

$$Q = V_B A = (30.29\text{ ft/s})\left[\pi\left(\frac{12}{12}\text{ft}\right)^2\right] = 95.16\text{ ft}^3/\text{s} = 95.2\text{ ft}^3/\text{s} \qquad \textbf{Ans.}$$

4-3.

Water flows out of a faucet at A at 6 m/s. Determine the velocity of the water just before it strikes the ground at B.



SOLUTION

If the datum is set at B, then $z_A = 1.75$ m and $z_B = 0$. Since the flow from A to B is in the atmosphere, $p_A = p_B = 0$. Applying Bernoulli's equation between A and B,

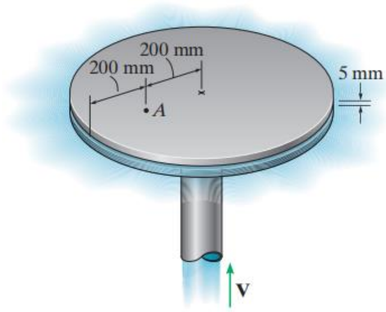
$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

$$0 + \frac{(6\ \text{m/s})^2}{2} + (9.81\ \text{m/s}^2)(1.75\ \text{m}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B = 8.387\ \text{m/s} = 8.39\ \text{m/s}$$ **Ans.**

4-4.

A fountain is produced by water that flows up the tube at 0.08 m^3/s, and then radially through two cylindrical plates before exiting to the atmosphere. Determine the velocity and pressure of the water at point A.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Writing this equation between points A and B on the radial streamline,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since point B is exposed to the atmosphere, $p_B = p_{atm} = 0$. Here, points A and B have the same elevation since the cylindrical plates are in the horizontal plane. Thus, $z_A = z_B = z$.

$$\frac{p_A}{1000\ \text{kg/m}^3} + \frac{V_A^2}{2} + gz = 0 + \frac{V_0^2}{2} + gz$$

$$p_A = 500\left(V_B^2 - V_A^2\right) \qquad \textbf{(1)}$$

Continuity requires that

$Q = V_A A_A;$ $\qquad 0.08\ \text{m}^3/\text{s} = V_A\,[2\pi(0.2\ \text{m})(0.005\ \text{m})]$

$$V_A = 12.73\ \text{m/s} = 12.7\ \text{m/s} \qquad \textbf{Ans.}$$

$Q = V_B A_B;$ $\qquad 0.08\ \text{m}^3/\text{s} = V_B[2\pi\,(0.4\ \text{m})(0.005\ \text{m})]$

$$V_B = 6.366\ \text{m/s}$$

Substituting these results into Eq. (1),

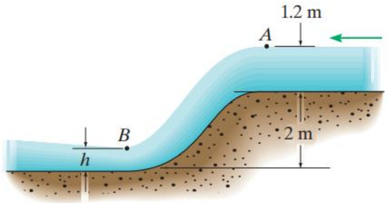
$$p_A = 500\left(6.366^2 - 12.73^2\right)$$

$$= -60.79\left(10^3\right)\ \text{Pa} = -60.8\ \text{kPa} \qquad \textbf{Ans.}$$

The negative sign indicates that the pressure at A is a partial vacuum.

4-5.

A river has an average width of 5 m. Just afte its flow falls 2 m to the lower elevation, the depth become $h = 0.8$ m. Determine the volumetric discharge.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between points A and B on the streamline along the water surface,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since the surface of water is exposed to the atmosphere, $p_A = p_B = p_{\text{atm}} = 0$ with reference to the datum set through point B, $z_A = 2\text{ m} + 1.2\text{ m} - 0.8\text{ m} = 2.4\text{ m}$ and $z_B = 0$.

$$0 + \frac{V_A^2}{2} + (9.81\text{ m/s}^2)(2.4\text{ m}) = 0 + \frac{V_B^2}{2} + 0$$

$$V_B^2 - V_A^2 = 47.088\text{ m}^2/\text{s}^2 \qquad \textbf{(1)}$$

Consider the fixed control volume that contains the water between the cross sections of the river through points A and B. Since the density of the water is constant and the average velocities will be used, the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A[5\text{ m}(1.2\text{ m})] + V_B[5\text{ m}(0.8\text{ m})] = 0$$

$$V_A = 0.6667 V_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$V_B = 9.206\text{ m/s} \qquad V_A = 6.138\text{ m/s}$$

Thus, the discharge is

$$Q = V_A A_A = (6.138\text{ m/s})[5\text{ m}(1.2\text{ m})] = 36.83\text{ m}^3/\text{s} = 36.8\text{ m}^3/\text{s} \qquad \textbf{Ans.}$$

4-6.

Air at a temperature of 40°C flows into the nozzle at 6 m/s and then exits to the atmosphere at B, where the temperature is 0°C. Determine the pressure at A.

SOLUTION

Assume that air is an ideal fluid (incompressible and inviscid) and the flow is steady. Then Bernoulli's equation is applicable. Writing this equation between points A and B on the central streamline,

$$\frac{p_a}{(\rho_a)_A} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{(\rho_a)_B} + \frac{V_B^2}{2} + gz_B$$

From Appendix A, $(\rho_a)_A = 1.127\ \mathrm{kg/m^3}$ ($T = 40°\mathrm{C}$) and $(\rho_a)_B = 1.292\ \mathrm{kg/m^3}$ ($T = 0°\mathrm{C}$). Since point B is exposed to the atmosphere $p_B = p_{\text{atm}} = 0$. Here the datum coincides with central streamline. Then $z_A = z_B = 0$.

$$\frac{p_A}{1.127\ \mathrm{kg/m^3}} + \frac{(6\ \mathrm{m/s})^2}{2} + 0 = 0 + \frac{V_B^2}{2} + 0$$

$$p_A = \left[0.5635\,(V_B^2 - 36)\right] p_a \qquad \textbf{(1)}$$

Consider the control volume to be the air within the nozzle. For steady flow, the continuity condition requires

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - (\rho_a)_A V_A A_A + (\rho_a)_B V_B A_B = 0$$

$$-\,(1.127\ \mathrm{kg/m^3})(6\ \mathrm{m/s})\left[\pi(0.15\ \mathrm{m})^2\right] + (1.292\ \mathrm{kg/m^3})(V_B)\left[\pi(0.05\ \mathrm{m})^2\right] = 0$$

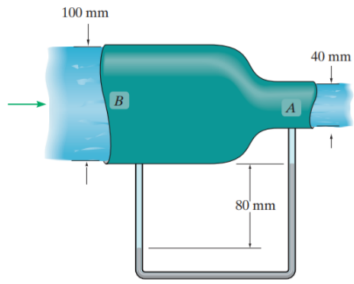
$$V_B = 47.10\ \mathrm{m/s}$$

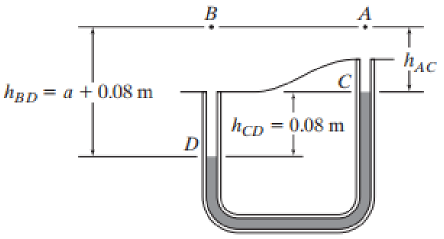
Substituting this result into Eq. 1,

$$\begin{aligned} p_A &= 0.5635\,(47.10^2 - 36) \\ &= 1229.98\ \mathrm{Pa} \\ &= 1.23\ \mathrm{kPa} \qquad \textbf{Ans.} \end{aligned}$$

4-7.

If the difference in the level of mercury within the manometer is 80 mm, determine the volumetric flow of the water. Take $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.





(a)

SOLUTION

Referring to Fig. *a*, the manometer equation written from *A* to *B* along the centerline is

$$p_A + \rho_w g h_{AC} + \rho_{Hg} g h_{CD} - \rho w g h_{BD} = p_B$$

$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h_{AC}) + (13550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.08\ \text{m})$$

$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h_{AC} + 0.08\ \text{m}) = p_B$$

$$p_B - p_A = 9849.24 \qquad \textbf{(1)}$$

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between points *A* and *B*,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + g z_A = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + g z_B$$

with reference to the datum set to coincide with the horizontal streamline connecting *A* and *B*, $z_A = z_B = 0$.

$$\frac{p_A}{1000\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{1000\ \text{kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$p_B - p_A = 500(V_A^2 - V_B^2) \qquad \textbf{(2)}$$

The fixed control volume considered contained the water within the transition. Since the density of the water is constant and the average velocities will be used, the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_B A_B + V_A A_A = 0$$

$$-V_B[\pi(0.05\ \text{m})^2] + V_A[\pi(0.02\ \text{m})^2] = 0$$

$$V_A = 6.25 V_B \qquad \textbf{(3)}$$

Substitute Eq. (1) into (2)

$$V_A^2 - V_B^2 = 19.69848 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4)

$$V_B = 0.7194\ \text{m/s} \qquad V_A = 4.496\ \text{m/s}$$

Then the discharge is

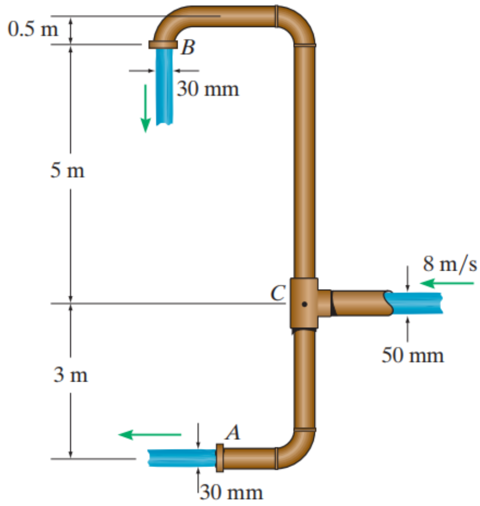
$$Q = V_B A_B = (0.7194\ \text{m/s})[\pi(0.05\ \text{m})^2]$$

$$= 0.005650\ \text{m}^3/\text{s}$$

$$= 0.00565\ \text{m}^3/\text{s} \qquad \textbf{Ans.}$$

4-8.

Determine the velocity out of the pipes at A and B if water flows into the Tee at 8 m/s and under a pressure of 40 kPa. The system is in the vertical plane.



SOLUTION

Continuity Equation. Consider the water within the pipe to be the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho\, \mathbf{V}\cdot d\mathbf{A} = 0$$

$$0 - V_C A_C + V_B A_B + V_A A_A = 0$$

$$-(8\text{ m/s})\left[\pi(0.025\text{ m})^2\right] + V_B\left[\pi(0.015\text{ m})^2\right] + V_A\left[\pi(0.015\text{ m})^2\right] = 0$$

$$V_A + V_B = 22.22 \qquad \textbf{(1)}$$

Bernoulli Equation. Since the water discharged into the atmosphere at A and B, $p_A = p_B = 0$. If we set the datum horizontally through point C, $z_B = 5$ m and $z_A = -3$ m.

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + g z_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + g z_A$$

$$0 + \frac{V_B^2}{2} + (9.81\text{ m/s}^2)(5\text{ m}) = 0 + \frac{V_A^2}{2} + (9.81\text{ m/s}^2)(-3\text{ m})$$

$$V_A^2 - V_B^2 = 156.96 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

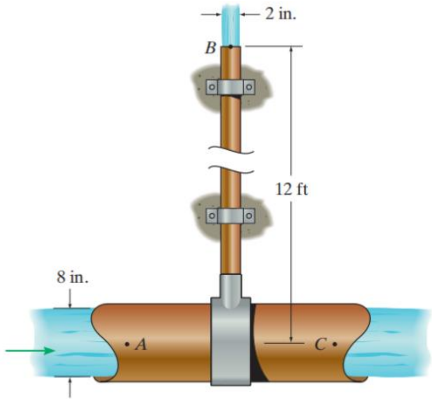
$$V_A = 14.6\text{ m/s} \qquad \textbf{Ans.}$$

$$V_B = 7.58\text{ m/s} \qquad \textbf{Ans.}$$

Note: Treating A and B as if they lie on the same streamline is a harmless shortcut. Officially, the solution process should proceed by considering *two* streamlines that each run through C

4-9.

Determine the velocity of water at B and C if the pressure of the water in the 8-in.-diameter pipe at A is 15 psi and the velocity at this point is 9 ft/s. The water is discharged into the atmosphere at B.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between A and B,

$$\frac{p_A}{\rho_w} + \frac{V_A{}^2}{2} + gz_A = \frac{p_B}{\rho_w} + \frac{V_B{}^2}{2} + gz_B$$

Since the water is discharged into the atmosphere at B, $p_B = 0$. With reference to the datum set through points A and C, $z_A = 0$ and $z_B = 12$ ft.

$$\frac{\left(15\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{\left(\frac{62.4\text{ l/ft}^3}{32.2\text{ ft/s}^2}\right)} + \frac{(9\text{ ft/s})^2}{2} + 0 = 0 + \frac{V_B^2}{2} + (32.2\text{ ft/s}^2)(12\text{ ft})$$

$$V_B = 39.21\text{ ft/s} = 39.2\text{ ft/s} \qquad \textbf{Ans.}$$

The fixed control volume considered contains the water within the pipe bounded by cross sections at A, B and C. Since the density of water is constant and the average velocities will be used, the continuity equation can be simplified as

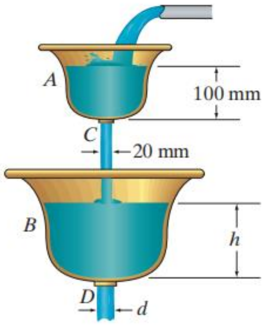
$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B + V_C A_C = 0$$

$$-(9\text{ ft/s})\left[\pi\left(\frac{4}{12}\text{ ft}\right)^2\right] + (39.21\text{ ft/s})\left[\pi\left(\frac{1}{12}\text{ ft}\right)^2\right] + V_C\left[\pi\left(\frac{4}{12}\text{ ft}\right)^2\right] = 0$$

$$V_C = 6.549\text{ ft/s} = 6.55\text{ ft/s} \qquad \textbf{Ans.}$$

4-10.



Water drains from the fountain cup A to cup B. If the depth in cup B is $h = 50$ mm, determine the velocity of the water at C and the diameter d of the opening at D so that steady flow is maintained.

SOLUTION

Since water can be considered as an ideal fluid (incompressible and inviscid) and the flow is required to be steady, Bernoulli's equation is applicable. Since A, B, C, and D are exposed to the atmosphere, $p_A = p_B = p_C = p_D = 0$. To maintain the steady flow, the level of water in cups A and B must be constant. Thus, $V_A = V_B = 0$. Between A and C with the datum at C, $z_C = 0$ and $z_A = 0.1$ m.

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho_w} + \frac{V_C^2}{2} + gz_C$$

$$0 + 0 + (9.81 \text{ m/s}^2)(0.1 \text{ m}) = 0 + \frac{V_C^2}{2} + 0$$

$$V_C = 1.401 \text{ m/s} = 1.40 \text{ m/s}$$ **Ans.**

Between B and D with the datum at D, $z_B = 0.05$ m and $z_D = 0$.

$$\frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B = \frac{p_D}{\rho_w} + \frac{V_D^2}{2} + gz_D$$

$$0 + 0 + (9.81 \text{ m/s}^2)(0.05 \text{ m}) = 0 + \frac{V_D^2}{2} + 0$$

$$V_D = 0.9904 \text{ m/s} = 0.990 \text{ m/s}$$

The fixed control volume that contains the water in cup B will be considered. Continuity requires that

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

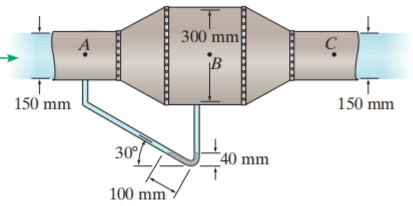
$$0 - V_C A_C + V_D A_D = 0$$

$$-(1.401 \text{ m/s})\left[\pi(0.01 \text{ m})^2\right] + (0.9904 \text{ m/s})\left(\frac{\pi}{4} d_D^2\right) = 0$$

$$d_D = 0.02378 \text{ m} = 23.8 \text{ mm}$$ **Ans.**

4-11.

Carbon dioxide at 20°C passes through the expansion chamber, which causes mercury in the manometer to settle as shown. Determine the velocity of the gas at A. Take $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



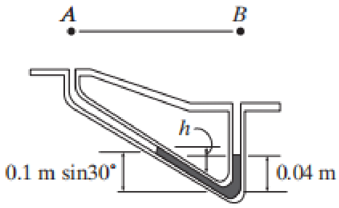
SOLUTION

Bernoulli Equation. From Appendix A, $\rho_{CO_2} = 1.84\ \text{kg/m}^2$ at $T = 20°\text{C}$. If we set the datum to coincide with the horizontal line connecting points A and B, $z_A = z_B = 0$.

$$\frac{p_A}{\rho_{CO_2}} + \frac{V_A^2}{2} + gz_A = \frac{p_B}{\rho_{CO_2}} + \frac{V_B^2}{2} + gz_B$$

$$\frac{p_A}{1.84\ \text{kg/m}^3} + \frac{V_A^2}{2} + 0 = \frac{p_B}{1.84\ \text{kg/m}^3} + \frac{V_B^2}{2} + 0$$

$$p_B - p_A = 0.920\left(V_A^2 - V_B^2\right) \qquad \textbf{(1)}$$



(a)

Continuity Equation. Consider the gas from A to B to the control volume.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi(0.075\ \text{m})^2\right] + V_B\left[\pi(0.15\ \text{m})^2\right] = 0$$

$$V_B = 0.25\, V_A \qquad \textbf{(2)}$$

Manometer Equation. Referring to Fig. a, $h = 0.1\ \text{m}\sin 30° - 0.04\ \text{m} = 0.01\ \text{m}$. Then, neglecting the weight of the CO_2,

$$p_A + \rho_{Hg} g h = p_B$$

$$p_A + \left(13\,550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.01\ \text{m}) = p_B$$

$$p_B - p_A = 1329.255 \qquad \textbf{(3)}$$

Equating Eqs. (1) and (3),

$$0.920\left(V_A^2 - V_B^2\right) = 1329.255$$

Substituting Eq. (2) into this equation,

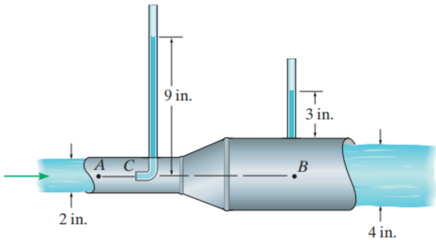
$$0.9375 V_A^2 = 1444.84$$

Thus,

$$V_A = 39.3\ \text{m/s} \qquad \textbf{Ans.}$$

4-12.

Determine the average velocity and the pressure in the pipe at A if the height of the water column in the pitot tube is 9 in. and the height in the piezometer is 3 in.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between points C and B.

$$\frac{p_C}{\rho_w} + \frac{V_C^2}{2} + gz_C = \frac{p_B}{\rho_w} + \frac{V_B^2}{2} + gz_B$$

Since point C is a stagnation point, $V_C = 0$, with reference to the datum set through the horizontal central streamline, $z_B = z_c = 0$. The pressures at points B and C are

$$p_B = \gamma_w h_B = (62.4\ \text{lb/ft}^3)\left(\frac{5}{12}\ \text{ft}\right) = 26\ \text{lb/ft}^2$$

$$p_C = \gamma_w h_C = (62.4\ \text{lb/ft}^3)\left(\frac{9}{12}\ \text{ft}\right) = 46.8\ \text{lb/ft}^2$$

Thus,

$$\frac{46.8\ \text{lb/ft}^2}{\left(\dfrac{62.4\ \text{lb/ft}^2}{32.2\ \text{ft/s}^2}\right)} + 0 + 0 = \frac{26\ \text{lb/ft}^2}{\left(\dfrac{62.4\ \text{lb/ft}^2}{32.2\ \text{ft/s}^2}\right)} + \frac{V_B^2}{2} + 0$$

$$V_B = 4.633\ \text{ft/s}.$$

The fixed control volume considered contains the water within the cross sections through A and B. Since the density of water is constant and the average velocities will be used, the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi\left(\frac{1}{12}\ \text{ft}\right)^2\right] + (4.633\ \text{ft/s})\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right] = 0$$

$$V_A = 18.53\ \text{ft/s} = 18.5\ \text{ft/s}$$ **Ans.**

Again, applying the Bernoulli's equation between points A and C,

$$\frac{p_A}{\rho_w} + \frac{V_A^2}{2} + gz_A = \frac{p_C}{\rho_w} + \frac{V_C^2}{2} + gz_C$$

With reference to the same datum, $z_A = z_C = 0$

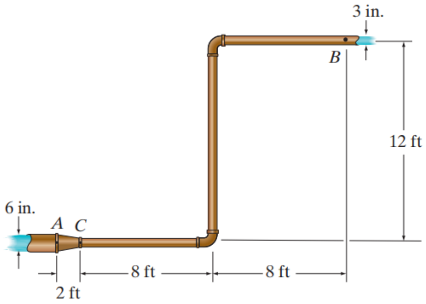
$$\frac{p_A}{\left(\dfrac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)} + \frac{(18.53\ \text{ft/s})^2}{2} + 0 = \frac{46.8\ \text{lb/ft}^2}{\left(\dfrac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}}\right)} + 0 + 0$$

$$p_A = (-286\ \text{lb/ft}^2)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = -1.99\ \text{psi}$$ **Ans.**

The negative sign indicates that p_A is **partial vacuum** or **suction.**

4-13.

Water at a pressure of 12 psi and a velocity of 5 ft/s at A flows through the transition. Plot the pressure head and the elevation head from A to B with reference to the datum set through A.



SOLUTION

The fixed control volume considered contained the water within the transition and the pipe between the cross section through A and B. Since the density of the water is constant and the average velocity will be used, the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A} = 0$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-(5\text{ ft/s})\left[\pi\left(\frac{3}{12}\text{ ft}\right)^2\right] + V_B\left[\pi\left(\frac{1.5}{12}\text{ ft}\right)^2\right] = 0$$

$$V_B = 20\text{ ft/s}$$

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Applying this equation between points A and B,

$$\frac{p_A}{\gamma_w} + \frac{{V_A}^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{{V_B}^2}{2g} + z_B$$

with reference to the datum set through point A, $z_A = 0$ and $z_B = 12$.

$$\frac{\left(12\,\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{62.4\text{ lb/ft}^3} + \frac{(5\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 = \frac{p_B}{62.4\text{ lb/ft}^3} + \frac{(20\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 12\text{ ft}$$

$$p_B = 615.85\text{ lb/ft}^2$$

Between A and C

$$\frac{p_A}{\gamma_w} + \frac{{V_A}^2}{2g} + z_A = \frac{p_C}{\gamma_w} + \frac{{V_C}^2}{2g} + z_C$$

Here $V_C = V_B = 20$ ft/s. Also, $z_C = z_A = 0$

$$\frac{\left(12\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\text{ in.}}{1\text{ ft}}\right)^2}{62.4\text{ lb/ft}^3} + \frac{(5\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0 = \frac{p_C}{62.4\text{ lb/ft}^3} + \frac{(20\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 0$$

$$p_C = 1364.65\text{ lb/ft}^2$$

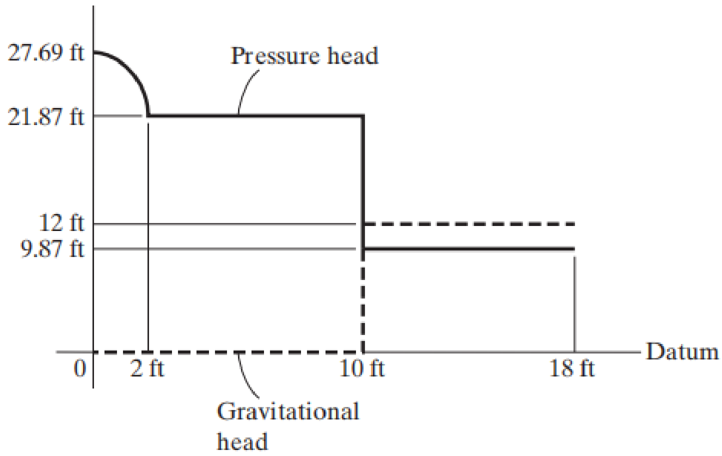
Therefore, the pressure head at A, B, and C are

$$\frac{p_A}{\gamma_w} = \frac{\left(12\dfrac{\text{lb}}{\text{in}^2}\right)\left(\dfrac{12\,\text{in.}}{1\,\text{ft}}\right)^2}{62.4\ \text{lb/ft}^3} = 27.69\ \text{ft}$$

$$\frac{p_B}{\gamma_w} = \frac{615.85\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} = 9.87\ \text{ft}$$

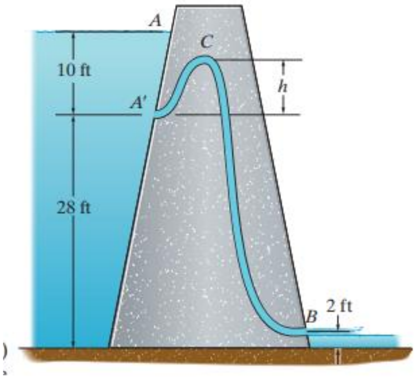
$$\frac{p_C}{\gamma_w} = \frac{1364.65\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} = 21.87\ \text{ft}$$

The gravitational head coincides with the centerline of the pipe. Plots of the pressure head and the gravitational head are shown in Fig. *a*.



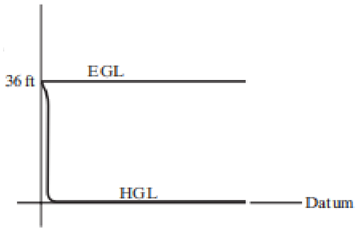
4-14.

The siphon spillway provides an automatic control of the level in the reservoir within a desired range. In the case shown, flow will begin when the water level in the reservoir rises above the crown C of the conduit. Determine the flow through the siphon if $h = 4$ ft. Also, draw the energy and hydraulic grade lines for the siphon conduit, with reference to the datum set through B. The siphon has a diameter of 8 in., and the water is at a temperature of 80°F. Neglect any head loss.



SOLUTION

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable. Since points A and B are exposed to the atmosphere, $p_A = p_B = p_{atm} = 0$. Also, the water is drawn from a large reservoir. Thus, the water level in the reservoir can be considered as constant and so $V_A = 0$. The height h of the crown will not affect the flow, provided the cavitation does not occur at C and $h < 10$ ft. with reference to the datum set through point B, $z_A = 38\text{ ft} - 2\text{ ft} = 36$ ft. and $z_B = 0$. Write the Bernoulli equation between A and B,



(a)

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$0 + 0 + 36\text{ ft} = 0 + \frac{V_B^2}{2(32.2\text{ ft/s}^2)} + 0$$

$$V_B = 48.15\text{ ft/s}$$

Using the result of $V_C = V_B = 48.15$ ft/s to write the Bernoulli equation between A and C with $z_A = 36$ ft and $z_C = 28\text{ ft} + 4\text{ ft} - 2\text{ ft} = 30$ ft.

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C$$

$$0 + 0 + 36\text{ ft} = \frac{p_C}{62.4\text{ lb/ft}^3} + \frac{(48.15\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} + 30\text{ ft}$$

$$p_C = \left(-1872\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = -13.0\text{ psi}$$

From the table in Appendix A, the vapor pressure for the water at 80°F is $(p_r)_{ab} = 0.507$ psia. Then its gage pressure is given by

$$(p_r)_{abs} = (p_r)_g + p_{atm}; \quad 0.507\text{ psi} = (p_r)_g + 14.7\text{ psi} \quad (p_r)_g = -14.19\text{ psi}$$

Since $p_r > (p_r)_g$, cavitation will not occur at C. Then the flow rate can be determined from

$$Q = V_B A = (48.15\text{ ft/s})\left[\pi\left(\frac{4}{12}\text{ ft}\right)^2\right] = 16.81\text{ ft}^3\text{/s} = 16.8\text{ ft}^3\text{/s} \qquad \textbf{Ans.}$$

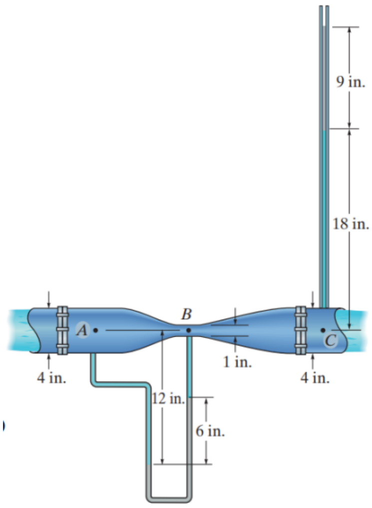
Since the diameter of the siphon conduit is constant, the velocity head is constant along the conduit:

$$\frac{V^2}{2g} = \frac{(48.15\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)} = 36\text{ ft.}$$

The plot of EGL and HGL with reference to the datum set through B is shown in Fig. a.

4-15.

A piezometer and a manometer containing mercury are connected to the venturi meter. If the levels are indicated, determine the volumetric flow of water through the meter. Draw the energy and hydraulic grade lines. Take $\gamma_{Hg} = 846\ \text{lb/ft}^3$.



SOLUTION

Referring to Fig. *a*, the manometer rule written from A to B gives

$$p_A + \gamma_w h_{AD} - \gamma_{Hg} h_{DE} - \gamma_w h_{BE} = p_B$$

$$p_A + (62.4\ \text{lb/ft}^3)(1\ \text{ft}) - (846\ \text{lb/ft}^3)(0.5\ \text{ft})(62.4\ \text{lb/ft}^3)(0.5\ \text{ft}) = p_B$$

$$p_A - p_B = 391.8 \qquad \textbf{(1)}$$

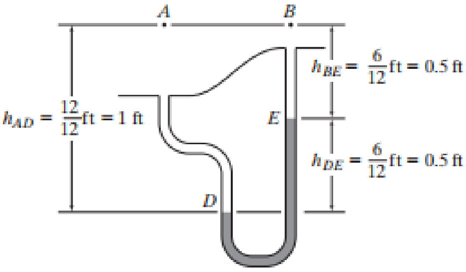
Also, from Fig. *b*, the manometer rule written from C to G gives

$$p_C - \gamma_w h_{CF} - \gamma_{Hg} h_{FG} = p_{atm} = 0$$

$$p_C - (62.4\ \text{lb/ft}^3)(1.5\ \text{ft}) - (846\ \text{lb/ft}^3)(0.75\ \text{ft}) = 0$$

$$p_C = 728.1\ \text{lb/ft}^2$$

Since the water can be considered as an ideal fluid (incompressible and inviscid) and the flow is steady, Bernoulli's equation is applicable with reference to the datum set through points, A and B, $z_A = z_B = 0$. Applying Bernoulli's equation between points A and B,



(a)

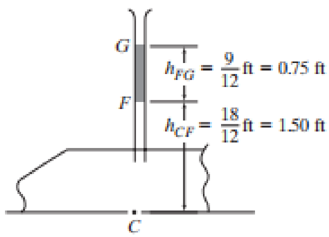
$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} + z_B$$

$$\frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} = \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g}$$

$$\frac{p_A - p_B}{\gamma_w} = \frac{1}{2g}(V_B^2 - V_A^2)$$

$$p_A - p_B = \frac{1}{2}\left(\frac{62.4\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(V_B^2 - V_A^2)$$

$$p_A - p_B = 0.9689(V_B^2 - V_A^2) \qquad \textbf{(2)}$$



(b)

The fixed control volume considered contains the water within the transition. Since the density of the water is constant and the average velocities will be used, the continuity equation can be simplified as

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\mathcal{V} + \int_{cs} \rho \mathbf{V} \cdot d\mathbf{A}$$

$$0 - V_A A_A + V_B A_B = 0$$

$$-V_A\left[\pi\left(\frac{2}{12}\text{ft}\right)^2\right] + V_B\left[\pi\left(\frac{0.5}{12}\text{ft}\right)^2\right] = 0$$

$$V_B = 16 V_A \qquad \textbf{(3)}$$

Substitute Eq. (1) into (2)

$$391.8 = 0.9689(V_B^2 - V_A^2)$$

$$V_B^2 - V_A^2 = 404.36 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4),

$$V_A = 1.2593 \text{ ft/s} \qquad V_B = 20.14 \text{ ft/s}$$

Then the flow rate is given by

$$Q = V_A A_A = (1.2593 \text{ ft/s})\left[\pi\left(\frac{2}{12}\text{ft}\right)^2\right] = 0.1099 \text{ ft}^3/\text{s} = 0.110 \text{ ft}^3/\text{s} \quad \textbf{Ans.}$$

The total energy head is

$$H = \frac{p_C}{\gamma_w} + \frac{V_C^2}{2g} + z_C = \frac{728.1 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} + \frac{(1.2593 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} + 0$$

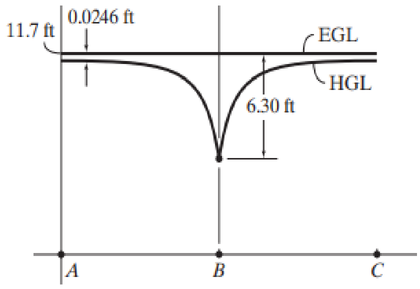
$$= 11.69 \text{ ft.}$$

The velocity heads at A (or C) and B are

$$\frac{V_C^2}{2g} = \frac{(1.2593 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 0.0246 \text{ ft}$$

$$\frac{V_B^2}{2g} = \frac{(20.14 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 6.30 \text{ ft}$$

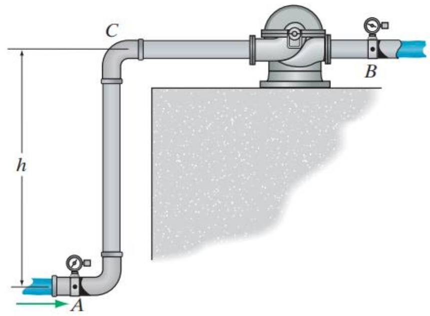
The energy and hydraulic grade lines with reference to the datum set through A, B, and C are plotted as shown in Fig. c



(c)

4-16.

Water is drawn into the pump, such that the pressure at the inlet A is $-6\ \text{lb/in}^2$ and the pressure at B is $20\ \text{lb/in}^2$. If the discharge at B is $4\ \text{ft}^3/\text{s}$ determine the power output of the pump. Neglect friction losses. The pipe has a constant diameter of 4 in. Take $h = 5$ ft and $\rho_w = 1.94\ \text{slug/ft}^3$.



SOLUTION

Energy Equation. Take the water from A to B to be the control volume. Since the pipe has a constant diameter, $V_A = V_B = V$. If we set the datum through A, $z_A = 0$ and $z_B = 5$ ft. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$\frac{-6\dfrac{\text{lb}}{\text{in}^2}\left(\dfrac{12\ \text{in.}}{\text{ft}}\right)^2}{1.94\dfrac{\text{slug}}{\text{ft}^3}\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)} + \frac{V^2}{2g} + 0 + h_{\text{pump}} = \frac{20\dfrac{\text{lb}}{\text{in}^2}\left(\dfrac{12\ \text{in.}}{\text{ft}}\right)^2}{1.94\dfrac{\text{slug}}{\text{ft}^3}\left(32.2\dfrac{\text{ft}}{\text{s}^2}\right)}$$

$$+ \frac{V^2}{2g} + 5\ \text{ft} + \text{h} + 0 + 0$$

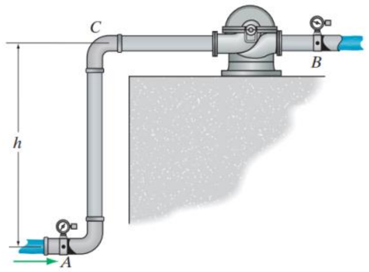
$$h_{\text{pump}} = 64.93$$

$$\begin{aligned}
\dot{W}_s &= Q_{\text{pump}}\,\gamma h_{\text{pump}} \\
&= (4\ \text{ft}^3/\text{s})(1.94\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(64.93\ \text{ft}) \\
&= 16225.36\ \text{ft.lb/s}\left(\frac{1\ \text{hp}}{550\ \text{ft.lb/s}}\right) \\
&= 29.5\ \text{hp}
\end{aligned}$$

Ans.

4-17.

Draw the energy and hydraulic grade lines for the pipe *ACB* in Prob. 5–89 using a datum at *A*.



SOLUTION

Discharge. Since the pipe has a constant diameter, the water velocity in the pipe is constant throughout the pipe as required by the continuity condition.

$$Q = VA;\quad 4\ \text{ft}^3/\text{s} = V\left[\pi\left(\frac{2}{12}\text{ft}\right)^2\right]$$

$$V = 45.84\ \text{ft/s}$$

Energy Equation. Take the water from A to B to be the control volume. With reference to the datum through A, $z_A = 0$ and $z_B = 5$ ft. With $h_L = 0$,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

$$= \frac{\left(-6\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2}{(1.94\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)} + \frac{V^2}{2g} + 0 + h_{\text{pum}} = \frac{\left(20\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2}{1.94\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)}$$

$$+ \frac{V^2}{2g} + 5\ \text{ft} + 0 + 0$$

$$h_{\text{pump}} = 64.93\ \text{ft}$$

EGL and HGL. Since no losses occur, the total head before the pump is

$$H = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A$$

$$\frac{\left(-6\frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2}{(1.94\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)} + \frac{(45.84\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 = 18.80\ \text{ft} = 18.8\ \text{ft}$$

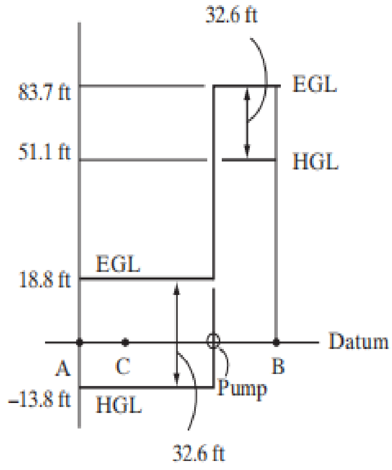
After the pump, a head of 64.93 ft is added to the water and becomes

$$H = 18.80\ \text{ft} + 64.93\ \text{ft} = 83.7\ \text{ft}$$

The velocity head has a constant value of

$$\frac{V^2}{2g} = \frac{(45.84\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 32.6\ \text{ft}$$

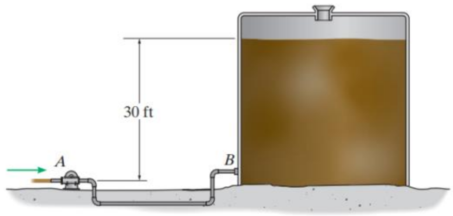
The HGL is always 32.6 ft below and parallel to the EGL. Both are plotted as shown in Fig. *a*.



(a)

4-18.

Crude oil is pumped from a test separator at A to the stock tank using a pipe that has a diameter of 4 in. If the total pipe length is 180 ft, and the volumetric flow at A is 400 gal/min, determine the horsepower supplied by the pump. The pressure at A is 4 psi, and the stock tank is open to the atmosphere. The frictional head loss in the pipe is 0.25 in./ft, the head loss at the pipe discharge into the tank is $1.0(V^2/2g)$, and for each of the four elbows it is $0.9(V^2/2g)$, and V is the velocity of the flow in the pipe. Take $\gamma_o = 55\ \text{lb/ft}^3$. Note that $1\ \text{ft}^3 = 7.48$ gal.



SOLUTION

The discharge is

$$Q = \left(400\ \frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 0.8913\ \text{ft}^3/\text{s}$$

Thus,

$$Q = VA; \qquad 0.8913\ \text{ft}^3/\text{s} = V\left[\pi\left(\frac{2}{12}\ \text{ft}\right)^2\right]$$

$$V = 10.21\ \text{ft/s}$$

Energy Equation.
Take the water from A to B as the control volume. Then

$$\frac{p_A}{\gamma_o} + \frac{V_A^2}{2g} + z_A + h_{\text{pump}} = \frac{p_B}{\gamma_o} + \frac{V_B^2}{2g} + z_B + h_{\text{turb}} + h_L$$

It is given that

$$p_A = \left(4\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 576\ \text{lb/ft}^2$$

The frictional head loss is

$$(h_L)_f = \left[\frac{(0.25/12)\ \text{ft}}{\text{ft}}\right](180\ \text{ft}) = 3.75\ \text{ft}$$

There are four elbows between points A and B. Thus, the head losses due to the elbows and the discharge into the tank is

$$(h_L)_M = 4\left(0.9\,\frac{V^2}{2g}\right) + 1.0\left(\frac{V^2}{2g}\right) = 4\left\{(0.9)\left[\frac{(10.21\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}\right]\right\}1.0\left[\frac{(10.21\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}\right] = 7.451\ \text{ft}$$

With reference to the datum set through point A, $z_A = 0$. On the outflow side, the pressure and elevation heads together add up to 30 ft, relative to the datum through point A. Substituting these results into the energy equation,

$$\frac{576\ \text{lb/ft}^2}{55.0\ \text{lb/ft}^3} + \frac{(10.21\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 + h_{\text{pump}} = \frac{(10.21\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 30\ \text{ft} + 0 + (3.75\ \text{ft} + 7.451\ \text{ft})$$

$$h_{\text{pump}} = 30.73\ \text{ft}$$

The required output power can be determined from

$$\dot{W}_s = Q\gamma_{co}h_{\text{pump}} = (0.8913\ \text{ft}^3/\text{s})(55.0\ \text{lb/ft}^3)(30.73\ \text{ft})$$

$$= (1506.27\ \text{ft}\cdot\text{lb/s})\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right)$$

$$= 2.74\ \text{hp} \qquad \textbf{Ans.}$$

Chapter5 Dimensional Analysis and Similarity

5-1.

During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the wave speed of an atomic bomb explosion. He assumed that the blast wave radius R was a function of energy released E, air density ρ, and time t. Use dimensional analysis to show how wave radius must vary with time.

SOLUTION

The proposed function is $R = f(E, \rho, t)$. There are four variables (n = 4) and three primary dimensions (MLT, or j = 3), thus we expect n-j = 4-3 = 1 pi group. List the dimensions:

$$\{R\} = \{L\} \;;\; \{E\} = \{ML^2/T^2\} \;;\; \{\rho\} = \{M/L^3\} \;;\; \{t\} = \{T\}$$

$$R^1 E^a \rho^b t^c = (L)^1 (ML^2/T^2)^a (M/L^3)^b (T)^c = M^0 L^0 T^0 ,$$

whence $a+b=0$; $1+2a-3b=0$; $-2a+c=0$; Solve $a=-\dfrac{1}{5}$; $b=+\dfrac{1}{5}$; $c=-\dfrac{2}{5}$

Assume arbitrary exponents and make the group dimensionless:

The single pi group is

$$\Pi_1 = \frac{R\,\rho^{1/5}}{E^{1/5}\,t^{2/5}} = \text{constant}, \quad \text{thus} \quad R_{wave} \propto t^{2/5} \qquad \textit{Ans.}$$

5-2.

In forced convection, the heat transfer coefficient h is a function of thermal conductivity k, density ρ, viscosity μ, specific heat c_p, body length L, and velocity V. Heat transfer coefficient has units of W/(m^2-K) and dimensions $\{MT^{-3}\Theta^{-1}\}$. Rewrite this relation in dimensionless form, using (k, ρ, c_p, L) as repeating variables.

SOLUTION

From Table 5.1, plus the given definition of h, list the dimensions:

h	k	ρ	μ	c_p	L	V
$\{MT^{-3}\Theta^{-1}\}$	$\{MLT^{-3}\Theta^{-1}\}$	$\{ML^{-3}\}$	$\{ML^{-1}T^{-1}\}$	$\{L^2T^{-2}\Theta^{-1}\}$	$\{L\}$	$\{LT^{-1}\}$

Four dimensions, 3 pi groups expected.

Add one variable successively to our repeating variables (k, ρ, c_p, L):

$$\Pi_1 = k^a \rho^b c_p^c L^d h^1 \quad \text{yields} \quad \Pi_1 = \frac{hL}{k}$$

$$\Pi_2 = k^a \rho^b c_p^c L^d \mu^1 \quad \text{yields} \quad \Pi_2 = \frac{\mu c_p}{k}$$

$$\Pi_3 = k^a \rho^b c_p^c L^d V^1 \quad \text{yields} \quad \Pi_3 = \frac{\rho L c_p V}{k}$$

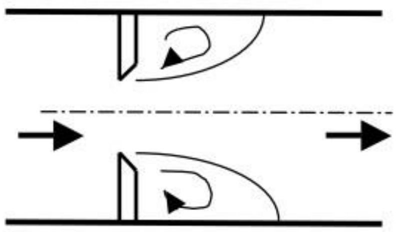
The final desired dimensionless function is

$$\frac{hL}{k} = fcn\left(\frac{\mu c_p}{k}, \frac{\rho L c_p V}{k}\right) \qquad \textit{Ans.}$$

In words, the Nusselt number is a function of Prandtl number and Peclet number.

5-3.

Flow in a pipe is often measured with an orifice plate, as in Fig. The volume flow Q is a function of the pressure drop Δp across the plate, the fluid density ρ, the pipe diameter D, and the orifice diameter d. Rewrite this functional relationship in dimensionless form.



SOLUTION

Write out the dimensions of the variables:

Q	Δp	ρ	D	d
$\{L^3/T\}$	$\{M/LT^2\}$	$\{M/L^3\}$	$\{L\}$	$\{L\}$

There are five variables (n = 5) and three dimensions (MLT), so we expect 5-3 = 2 Pi groups. This can almost be done by inspection, although using (Δp, ρ,D) as repeating variables will also work fine. Only two, Q and Δp, contain time $\{T\}$, so we must divide $Q/\Delta p^{1/2}$, with dimensions $\{L^{7/2}/M^{1/2}\}$, to eliminate T. Then, to eliminate $\{M^{1/2}\}$, we multiply by $\rho^{1/2}$, giving $\rho Q/\Delta p^{1/2}$, with dimensions $\{L^2\}$. We finish by dividing by D^2. That is Π_1, and Π_2 is simply d/D. The final dimensionless function is

$$\frac{\rho Q}{D^2\sqrt{\Delta p}} = fcn\left(\frac{d}{D}\right) \qquad \textit{Ans.}$$

5-4.

Under laminar conditions, the volume flow Q through a small triangular-section pore of side length b and length L is a function of viscosity μ, pressure drop per unit length $\Delta p/L$, and b. Using the pi theorem, rewrite this relation in dimensionless form. How does the volume flow change if the pore size b is doubled?

SOLUTION

Establish the variables and their dimensions:

$$Q = \text{fcn}(\Delta p/L, \mu, b)$$

$$\{L^3/T\} \quad \{M/L^2T^2\} \quad \{M/LT\} \quad \{L\}$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 = \mathbf{1}$ Pi group, found as follows:

$$\Pi_1 = (\Delta p/L)^a (\mu)^b (b)^c Q^1 = \{M/L^2T^2\}^a \{M/LT\}^b \{L\}^c \{L^3/T\}^1 = M^0L^0T^0$$

$$M\text{:}\ a + b = 0; \quad L\text{:}\ -2a - b + c + 3 = 0; \quad T\text{:}\ -2a - b - 1 = 0,$$

$$\text{solve} \quad a = -1,\ b = +1,\ c = -4$$

$$\Pi_1 = \frac{\mathbf{Q}\mu}{(\mathbf{\Delta p/L})\mathbf{b}^4} = \textbf{constant} \quad \textit{Ans.}$$

Clearly, if b is doubled, the flow rate Q increases by a factor of $2^4 =$ **<u>16</u>**. *Ans.*

5-5.

A pendulum has an oscillation period *T* which is assumed to depend upon its length *L*, bob mass *m*, angle of swing θ, and the acceleration of gravity. A pendulum 1 m long, with a bob mass of 200 g, is tested on earth and found to have a period of 2.04 s when swinging at 20°. (a) What is its period when it swings at 45°? A similarly constructed pendulum, with L = 30 cm and m = 100 g, is to swing on the moon (g = 1.62 m/s^2) at θ = 20°. (b) What will be its period?

SOLUTION

First establish the variables and their dimensions so that we can do the numbers:

$$T = \text{fcn}(\; L \;,\; m \;,\; g \;,\; \theta\;)$$

$$\{T\} \qquad \{L\} \quad \{M\} \quad \{L/T^2\} \quad \{1\}$$

Then n = 5 and j = 3, hence we expect n – j = 5 – 3 = **2** Pi groups. They are unique:

$$T\sqrt{\frac{g}{L}} = \text{fcn}(\theta) \quad \text{(mass drops out for dimensional reasons)}$$

(a) If we change the angle to 45°, this changes Π_2, hence we lose dynamic similarity and **do not know the new period**. More testing is required. *Ans.* (a)

(b) If we swing the pendulum on the moon at the same 20°, we may use similarity:

$$T_1\left(\frac{g_1}{L_1}\right)^{1/2} = (2.04\text{ s})\left(\frac{9.81\text{ m/s}^2}{1.0\text{ m}}\right)^{1/2} = 6.39 = T_2\left(\frac{1.62\text{ m/s}^2}{0.3\text{ m}}\right)^{1/2},$$

$$\text{or:} \quad \mathbf{T_2 = 2.75\ s} \quad \textit{Ans.}\text{(b)}$$

5-6.

When fluid in a long pipe starts up from rest at a uniform acceleration a, the initial flow is laminar. The flow undergoes transition to turbulence at a time t^* which depends, to first approximation, only upon a, ρ, and μ. Experiments by P. J. Lefebvre, on water at 20°C starting from rest with 1-g acceleration in a 3-cm-diameter pipe, showed transition at $t^* = 1.02$ s. Use this data to estimate (a) the transition time, and (b) the transition Reynolds number Re_D for water flow accelerating at 35 m/s^2 in a 5-cm-diameter pipe.

SOLUTION

For water at 20°C, take $\rho = 998$ kg/m^3 and $m = 0.001$ kg/m-s. There are four variables. Write out their dimensions:

t^*	a	ρ	μ
$\{T\}$	$\{LT^{-2}\}$	$\{ML^{-3}\}$	$\{ML^{-1}T^{-1}\}$

There are three primary dimensions, (MLT), hence we expect 4 – 3 = one pi group:

$$\Pi_1 = \rho^a \mu^b a^c t^{*1} \quad \text{yields} \quad \Pi_1 = t^*\left(\frac{\rho a^2}{\mu}\right)^{1/3}, \quad or \quad t^* = (const)\left(\frac{\mu}{\rho a^2}\right)^{1/3}$$

Use LeFebvre's data point to establish the constant value of Π_1:

$$t^* = 1.02 = (const)\left[\frac{0.001\,kg/m-s}{(998\,kg/m^3)(9.81\,m/s^2)^2}\right]^{1/3} = (const)(0.00218)$$

Thus the constant, or Π_1, equals 1.02/0.00218 = 467 (dimensionless). Use this value to establish the new transition time for $a = 35$ m/s^2 in a 5-cm-diameter pipe:

$$t^* = (467)\left(\frac{\mu}{\rho a^2}\right)^{1/3} = (467)\left[\frac{0.001}{998(35)^2}\right]^{1/3} = \mathbf{0.44}\,s \qquad Ans.(a)$$

$$Re_D = \frac{\rho VD}{\mu} = \frac{\rho(aT^*)D}{\mu} = \frac{998[35(0.44)](0.05)}{(0.001)} = \mathbf{768{,}000} \qquad Ans.(b)$$

This transition Reynolds number is more than 300 times the value for which *steady* laminar pipe flow undergoes transition. The reason is that this is a thin-boundary-layer flow, and the laminar velocity profile never even approaches the Poiseuille parabola.

Chapter 6 • Viscous Flow in Ducts

6-1.

The Keystone Pipeline in the chapter opener photo has a maximum proposed flow rate of 1.3 million barrels of crude oil per day. Estimate the Reynolds number and whether the flow is laminar. Assume that Keystone crude oil fits Fig. A.1 of the Appendix at 40°C.

SOLUTION

From Fig. A.1 of the Appendix, for crude oil at 40°C, ρ = (SG)ρ_{water} = 0.86(1000) = 860 kg/m^3, and $\mu \approx 0.0054$ kg/m·s. (a) Convert 1,300,000 barrels per day to 2.39 m^3/s (Appendix C) and a diameter of 36 in equals 0.914 m. Then the Reynolds number is

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi \mu d} = \frac{4(860 kg/m^3)(2.39 m^3/s)}{\pi(0.0054 kg/m \cdot s)(0.914 m)} = 530{,}000$$

The flow is definitely *turbulent*. *Ans.*

6-2.

SAE 10W30 oil at 20ºC flows from a tank into a 2 cm-diameter tube 40 cm long. The flow rate is 1.1 m^3/hr. Is the entrance length region a significant part of this tube flow?

SOLUTION

From Table A.4, for SAE 10W30 oil, read ρ = 876 kg/m^3 and μ = 0.17 kg/m·s. The flow rate is (1.1 m^3/hr)/(3600 s/hr) = 0.0003056 m^3/s, and the average velocity is $V = 4Q/(\pi d^2)$ = 0.973 m/s. The Reynolds number and entrance length are thus

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{(876)(0.973)(0.02)}{0.17} = 100 \quad (laminar)$$

$$\text{From Eq.(6.5)}, \; L_e \approx (0.06\,\mathrm{Re}_d)\,d = (0.06)(100)(2\,cm) \approx 12\ cm$$

This is 30% of the total tube length, so it is indeed significant, and entrance loss should be included.

6-3.

Professor Gordon Holloway and his students at the University of New Brunswick went to a fast-food emporium and tried to drink chocolate shakes ($\rho \approx$ 1200 kg/m^3, $\mu \approx$ 6 kg/m·s) through fat straws 8 mm in diameter and 30 cm long. (a) Verify that their human lungs, which can develop approximately 3000 Pa of vacuum pressure, would be unable to drink the milkshake through the vertical straw. (b) A student cut 15 cm from his straw and proceeded to drink happily. What rate of milkshake flow was produced by this strategy?

SOLUTION

(a) Assume the straw is barely inserted into the milkshake. Then the energy equation predicts

$$\frac{p_1}{\rho g} = \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} = \frac{V_2^2}{2g} + z_2 + h_f$$

$$= 0 + 0 + 0 = \frac{(-3000\ Pa)}{(1200\ kg/m^3)(9.81\ m/s^2)} + \frac{V_{tube}^2}{2g} + 0.3\ m + h_f$$

$$Solve\ for\ \mathbf{h_f = 0.255\ m - 0.3\ m - \frac{V_{tube}^2}{2g} < 0} \quad \textit{\textbf{which is impossible}} \quad Ans.(a)$$

(b) By cutting off 15 cm of vertical length and assuming laminar flow, we obtain a new energy equation

$$h_f = 0.255 - 0.15 - \frac{V^2}{2g} = \frac{32\mu LV}{\rho g d^2} = 0.105\ m - \frac{V^2}{2(9.81)} = \frac{32(6.0)(0.15)V}{(1200)(9.81)(0.008)^2} = 38.23V$$

Solve for $\quad V = 0.00275\ m/s, \quad Q = AV = (\pi/4)(0.008)^2(0.00275)$

$$Q = 1.4E{-}7\ \frac{m^3}{s} = \mathbf{0.14\ \frac{cm^3}{s}} \quad Ans.\ (b)$$

Check the Reynolds number: $\mathrm{Re_d} = \rho Vd/\mu = (1200)(0.00275)(0.008)/(6) = 0.0044$ (Laminar).

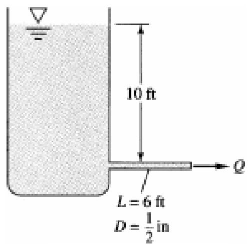
6-4.

An oil (SG = 0.9) issues from the pipe in Fig. at Q = 35 ft^3/h. What is the kinematic viscosity of the oil in ft^3/s? Is the flow laminar?

SOLUTION

Apply steady-flow energy:

$$\frac{p_{atm}}{\rho g} + \frac{0^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f,$$



where $V_2 = \frac{Q}{A} = \frac{35/3600}{\pi(0.25/12)^2} \approx 7.13\ \frac{ft}{s}$

Solve $h_f = z_1 - z_2 - \frac{V_2^2}{2g} = 10 - \frac{(7.13)^2}{2(32.2)} = 9.21$ ft

Assuming laminar pipe flow, use Eq. (6.12) to relate head loss to viscosity:

$$h_f = 9.21 \text{ ft} = \frac{128\nu LQ}{\pi g d^4} = \frac{128(6)(35/3600)\nu}{\pi(32.2)(0.5/12)^4}, \quad \text{solve } \nu = \frac{\mu}{\rho} \approx \mathbf{3.76E{-}4}\ \frac{\mathbf{ft^2}}{\mathbf{s}} \quad Ans.$$

Check Re = $4Q/(\pi\nu d) = 4(35/3600)/[\pi(3.76E{-}4)(0.5/12)] \approx 790$ (OK, laminar)

6-5.

Two tanks of water at 20°C are connected by a capillary tube 4 mm in diameter and 3.5 m long. The surface of tank 1 is 30 cm higher than the surface of tank 2. (a) Estimate the flow rate in m^3/h. Is the flow laminar? (b) For what tube diameter will Re_d be 500?

SOLUTION

For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. (a) Both tank surfaces are at atmospheric pressure and have negligible velocity. The energy equation, when neglecting minor losses, reduces to:

$$\Delta z = 0.3\ m = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)(0.004\ m)^4}$$

$$\text{Solve for}\quad Q = 5.3E\text{–}6\ \frac{m^3}{s} = \mathbf{0.019\ \frac{m^3}{h}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{Check}\quad \mathrm{Re_d} = 4\rho Q/(\pi\mu d) = 4(998)(5.3\mathrm{E}\text{–}6)/[\pi(0.001)(0.004)]$$

$$\mathbf{Re_d = 1675\ \textit{laminar.}} \quad \textit{Ans.}\text{ (a)}$$

(b) If $Re_d = 500 = 4\rho Q/(\pi\mu d)$ and $\Delta z = h_f$, we can solve for both Q and d:

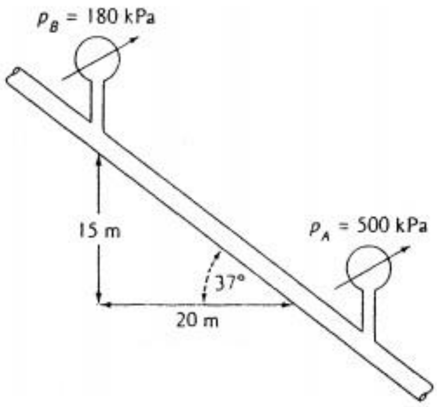
$$\mathrm{Re}_d = 500 = \frac{4(998\ kg/m^3)Q}{\pi(0.001\ kg/m{\cdot}s)d}, \quad \textbf{\textit{or}} \quad Q = 0.000394d$$

$$h_f = 0.3\ m = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4}, \quad or \quad Q = 20600d^4$$

Combine these two to solve for $Q = 1.05\ E\text{-}6\ m^3/s$ *and* **d = 2.67 mm** *Ans.*(b)

6-6.

SAE 30 oil at 20°C flows in the 3- cm-diameter pipe in Fig. , which slopes at 37°. For the pressure measurements shown, determine (a) whether the flow is up or down and (b) the flow rate in m^3/h.



SOLUTION

For SAE 30 oil, take $\rho = 891$ kg/m^3 and $\mu = 0.29$ kg/m·s. Evaluate the hydraulic grade lines:

$$\mathrm{HGL_B} = \frac{p_B}{\rho g} + z_B = \frac{180000}{891(9.81)} + 15 = 35.6 \text{ m}; \quad \mathrm{HGL_A} = \frac{500000}{891(9.81)} + 0 = 57.2 \text{ m}$$

$$\text{Since} \quad \mathrm{HGL_A} > \mathrm{HGL_B} \text{ the } \textbf{flow is } \boldsymbol{up} \quad \textit{Ans.}\text{(a)}$$

The head loss is the difference between hydraulic grade levels:

$$h_f = 57.2 - 35.6 = 21.6 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.29)(25)Q}{\pi(891)(9.81)(0.03)^4}$$

$$\text{Solve for} \quad Q = 0.000518 \text{ m}^3/\text{s} \approx \mathbf{1.86\ m^3/h} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Finally, check} \quad \mathrm{Re} = 4\rho Q/(\pi\mu d) \approx 68 \text{ (OK, laminar flow).}$$

6-7.

Water at 20°C is pumped from a reservoir through a vertical tube 10 ft long and 1/16th inch in diameter. The pump provides a pressure rise of 11 lbf/in^2 to the flow. Neglect entrance losses. (*a*) Calculate the exit velocity. (*b*) Approximately how high will the exit water jet rise? (*c*) Verify that the flow is laminar.

SOLUTION

For water at 20°C, Table A.3, $\rho = 998$ kg/m^3 = 1.94 slug/ft^3, and $\mu = 0.001$ kg/m-s = 2.09E-5 slug/ft-s. The energy equation, with 1 at the bottom and 2 at the top of the tube, is:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{11(144)}{1.94(32.2)} + 0 + 0 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f = 0 + \frac{V_2^2}{2g} + 10 + \frac{32\mu L V_2}{\rho g D^2}$$

$$\text{or: } 25.4 = \frac{V_{exit}^2}{2(32.2)} + 10 + \frac{32(0.0000209)(10)V}{(1.94)(32.2)(0.00521)^2}; \text{ or: } 15.4\ ft = \frac{V_{exit}^2}{64.4} + 3.94\ V_{exit}$$

(*a, c*) The velocity head is very small (<1 ft), so the dominant term is 3.94 V_{exit}. One can easily iterate, or simply use Excel to find the result:

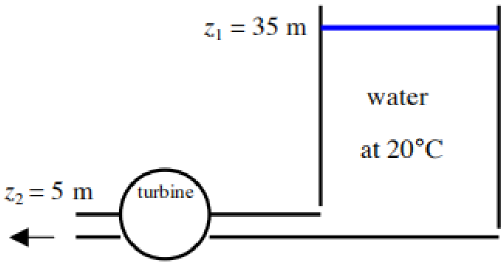
$$V_{exit} = 3.84\ \frac{ft}{s} \quad Ans.(a) \quad ; \quad \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{(1.94)(3.84)(0.00521)}{0.0000209} = \mathbf{1860\ \ laminar} \quad Ans.(c)$$

(*b*) Assuming frictionless flow outside the tube, the jet would rise due to the velocity head:

$$H_{rise} = \frac{V_{exit}^2}{2g} = \frac{(3.84\ ft/s)^2}{2(32.2\ ft/s^2)} = 0.229\ ft \approx \mathbf{2.75}\ \ inches \qquad Ans.(b)$$

6-8.

A reservoir supplies water through 100 m of 30-cm-diameter cast iron pipe to a turbine that extracts 80 hp from the flow. The water then exhausts to the atmosphere. Neglect minor losses. (*a*) Assuming that $f \approx 0.019$, find the flow rate (there is a cubic polynomial). Explain why there are two solutions.



(*b*) For extra credit, solve for the flow rate using the actual friction factors.

SOLUTION

For water at 20°C, take ρ = 998 kg/m³ and μ = 0.001 kg/m-s. The energy equation yields a relation between elevation, friction, and turbine power:

$$\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_2}{\rho g}} + \frac{V_2^2}{2g} + z_2 + h_{turb} + h_f$$

$$z_1 - z_2 = 35 - 5m = 30m = h_{turb} + h_f = \frac{Power}{\rho g Q} + (1 + f\frac{L}{D})\frac{V_2^2}{2g}, \quad Q = \frac{\pi}{4}D^2 V_2$$

$$30\,m = \frac{(80\,hp)(745.7W/hp)}{(9790\,N/m^3)(\pi/4)(0.3m)^2 V_2} + [1 + (0.019)\frac{100m}{0.3\,m}]\frac{V_2^2}{2(9.81m/s^2)}$$

Clean this up into a cubic polynomial:

$$30 = \frac{86.2}{V} + 0.373V^2, \quad \text{or:} \quad V^3 - 80.3V + 231 = 0$$

$$\textit{Three roots:} \quad V = 3.34\,m/s \;;\; 6.81\,m/s \;;\; -10.15\,m/s$$

The third (negative) root is meaningless. The other two are correct. Either

Q = 0.481 m³/s , $h_{turbine}$ = 12.7 m , h_f = 17.3 m

Q = 0.236 m³/s , $h_{turbine}$ = 25.8 m , h_f = 4.2 m *Ans.*(*a*)

Both solutions are valid. The higher flow rate wastes a lot of water and creates 17 meters of friction loss. The lower rate uses 51% less water and has proportionately much less friction.

(*b*) The *actual* friction factors are very close to the problem's "Guess". Thus we obtain

Re = 2.04E6, f = 0.0191; **Q = 0.479 m³/s** , $h_{turbine}$ = 12.7 m , h_f = 17.3 m

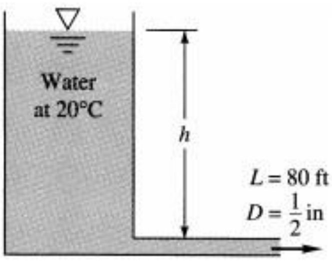
Re = 1.01E6, f = 0.0193 ; **Q = 0.237 m³/s** , $h_{turbine}$ = 25.7 m , h_f = 4.3 m

Ans.(b)

The same remarks apply: The lower flow rate is better, less friction, less water used.

6-9.

What level h must be maintained in Fig. to deliver a flow rate of 0.015 ft^3/s through the $\frac{1}{2}$-in commercial-steel pipe?



SOLUTION

For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft, or $\varepsilon/d = 0.00015/(0.5/12) \approx 0.0036$. Compute

$$V = \frac{Q}{A} = \frac{0.015}{(\pi/4)(0.5/12)^2} = 11.0\ \frac{ft}{s};$$

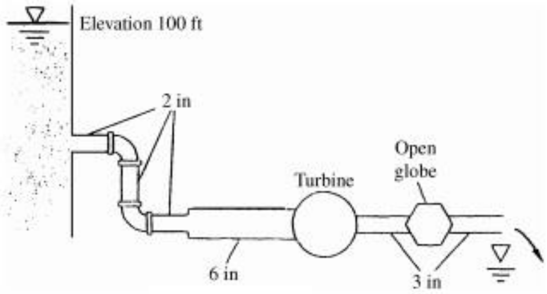
$$Re = \frac{\rho V d}{\mu} = \frac{1.94(11.0)(0.5/12)}{2.09E{-}5} \approx 42500 \quad \varepsilon/d = 0.0036, \quad f_{Moody} \approx 0.0301$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx 0$, yields an expression for surface elevation:

$$h = h_f + \frac{V^2}{2g} = \frac{V^2}{2g}\left(1 + f\frac{L}{d}\right) = \frac{(11.0)^2}{2(32.2)}\left[1 + 0.0301\left(\frac{80}{0.5/12}\right)\right] \approx \mathbf{111\ ft} \quad \textit{Ans.}$$

6-10.

In Fig. there are 125 ft of 2-in pipe, 75 ft of 6-in pipe, and 150 ft of 3-in pipe, all cast iron. There are three 90° elbows and an open globe valve, all flanged. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 0.16 ft^3/s of water at 20°C?



SOLUTION

For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. The 2″, 6″, and 3″ pipes have, respectively,

$$\text{(a)}\quad L/d = 750,\ \varepsilon/d = 0.0051;\quad \text{(b)}\quad L/d = 150,\ \varepsilon/d = 0.0017;$$

$$\text{(c)}\quad L/d = 600,\ \varepsilon/d = 0.0034$$

The flow rate is known, so each velocity, Reynolds number, and f can be calculated:

$$V_a = \frac{0.16}{\pi(2/12)^2/4} = 7.33\ \frac{\text{ft}}{\text{s}};\quad Re_a = \frac{1.94(7.33)(2/12)}{2.09E{-}5} = 113500,\quad f_a \approx 0.0314$$

$$\text{Also, } V_b = 0.82 \text{ ft/s},\ Re_b = 37800,\ f_c \approx 0.0266;\ V_c = 3.26,\ Re_c = 75600,\ f_c \approx 0.0287$$

Finally, the minor loss coefficients may be tabulated:

$$\text{sharp 2″ entrance: } K = 0.5;\quad \text{three 2″ 90° elbows: } K = 3(0.95)$$

$$\text{2″ sudden expansion: } K \approx 0.79;\quad \text{3″ open globe valve: } K \approx 6.3$$

The turbine head equals the elevation difference minus losses and the exit velocity head:

$$h_t = \Delta z - \sum h_f - \sum h_m - V_c^2/(2g)$$

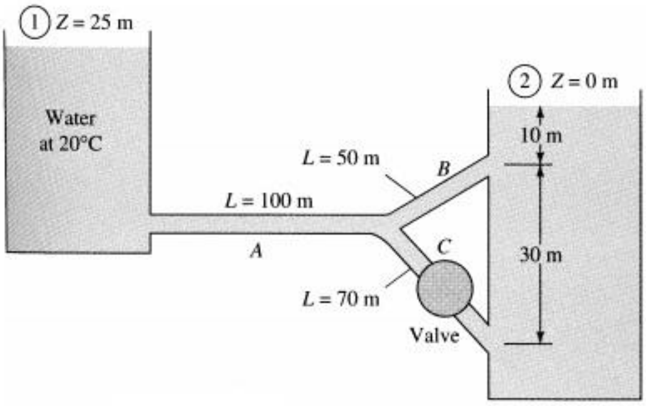
$$= 100 - \frac{(7.33)^2}{2(32.2)}[0.0314(750) + 0.5 + 3(0.95) + 0.79]$$

$$- \frac{(0.82)^2}{2(32.2)}(0.0266)(150) - \frac{(3.26)^2}{2(32.2)}[0.0287(600) + 6.3 + 1] \approx \mathbf{72.8\ ft}$$

The resulting turbine power = $\rho g Q h_t$ = (62.4)(0.16)(72.8) ÷ 550 ≈ **1.32 hp**. *Ans.*

6-11.

In Fig. all pipes are 8-cm-diameter cast iron. Determine the flow rate reservoir (1) if valve C is (a) closed; and (b) open, with $K_{valve} = 0.5$.



SOLUTION

For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence $\varepsilon/d = 0.26/80 \approx 0.00325$ for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

sharp entrance at A: $K_1 \approx 0.5$; line junction from A to B: $K_2 \approx 0.9$ (Table 6.5)

branch junction from A to C: $K_3 \approx 1.3$; two submerged exits: $K_B = K_C \approx 1.0$

If valve C is closed, we have a straight *series* path through A and B, with the same flow rate Q, velocity V, and friction factor *f* in each. The energy equation yields

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB},$$

$$\text{or:}\quad 25\text{ m} = \frac{V^2}{2(9.81)}\left[f\frac{100}{0.08} + 0.5 + 0.9 + f\frac{50}{0.08} + 1.0\right],\quad \text{where } f = \text{fcn}\left(\text{Re}, \frac{\varepsilon}{d}\right)$$

Guess $f \approx f_{\text{fully rough}} \approx 0.027$, then $V \approx 3.04$ m/s, Re $\approx 998(3.04)(0.08)/(0.001) \approx 243000$, $\varepsilon/d = 0.00325$, then $f \approx 0.0273$ (converged). Then the velocity through A and B is V = 3.03 m/s, and $Q = (\pi/4)(0.08)^2(3.03) \approx$ **0.0152 m³/s**. *Ans.* (a).

If valve C is open, we have parallel flow through B and C, with $Q_A = Q_B + Q_C$ and, with *d* constant, $V_A = V_B + V_C$. The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \sum h_{mA\text{-}B} + h_{fB} + \sum h_{mB} = h_{fA} + \sum h_{mA\text{-}C} + h_{fC} + \sum h_{mC},$$

$$\text{or:}\quad 25 = \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08}+0.5+0.9\right]+\frac{V_B^2}{2(9.81)}\left[f_B\frac{50}{0.08}+1.0\right]$$

$$= \frac{V_A^2}{2(9.81)}\left[f_A\frac{100}{0.08}+0.5+1.3\right]+\frac{V_C^2}{2(9.81)}\left[f_C\frac{70}{0.08}+1.0\right]$$

plus the additional relation $V_A = V_B + V_C$. Guess $f \approx f_{\text{fully rough}} \approx 0.027$ for all three pipes and begin. The initial numbers work out to

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

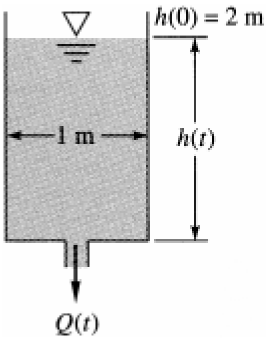
$$\text{If } f \approx 0.027,\ \text{solve (laboriously) } V_A \approx 3.48\text{ m/s}, V_B \approx 1.91\text{ m/s}, V_C \approx 1.57\text{ m/s.}$$

$$\text{Compute } Re_A = 278000,\quad f_A \approx 0.0272,\quad Re_B = 153000,\quad f_B = 0.0276,$$
$$Re_C = 125000,\quad f_C = 0.0278$$

Repeat once for convergence: $V_A \approx 3.46$ m/s, $V_B \approx 1.90$ m/s, $V_C \approx 1.56$ m/s. The flow rate from reservoir (1) is $\mathbf{Q_A} = (\pi/4)(0.08)^2(3.46) \approx \mathbf{0.0174\ m^3/s}$. (14% more) *Ans.* (b)

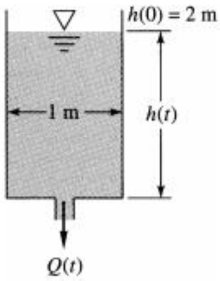
6-12.

The 1-m-diameter tank in Fig. is initially filled with gasoline at 20°C. There is a 2-cm-diameter orifice in the bottom. If the orifice is suddenly opened, estimate the time for the fluid level $h(t)$ to drop from 2.0 to 1.6 meters.



SOLUTION

For gasoline at 20°C, take $\rho = 680$ kg/m^3 and μ = 2.92E−4 kg/m·s. The



orifice simulates "comer taps" with $\beta \approx 0$, so, from Eq. (6.112), $C_d \approx 0.596$. From the energy equation, the pressure drop across the orifice is $\Delta p = \rho g h(t)$, or

$$Q = C_d A_t \sqrt{\frac{2\rho g h}{\rho(1-\beta^4)}} \approx 0.596\left(\frac{\pi}{4}\right)(0.02)^2\sqrt{2(9.81)h} \approx 0.000829\sqrt{h}$$

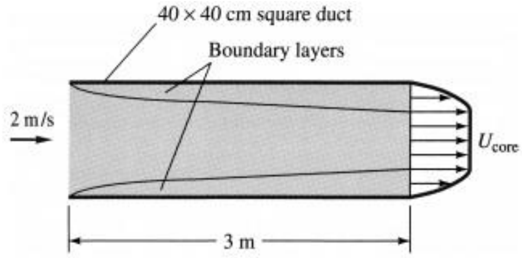
$$\text{But also}\quad Q = -\frac{d}{dt}(\upsilon_{tank}) = -A_{tank}\frac{dh}{dt} = -\frac{\pi}{4}(1.0\text{ m})^2\frac{dh}{dt}$$

Set the Q's equal, separate the variables, and integrate to find the draining time:

$$-\int_{2.0}^{1.6}\frac{dh}{\sqrt{h}} = 0.001056\int_0^{t_{final}} dt, \quad \text{or} \quad t_{final} = \frac{2[\sqrt{2}-\sqrt{1.6}]}{0.001056} = 283\text{ s} \approx \mathbf{4.7\ min} \quad \textit{Ans.}$$

6-13.

Air at 20°C and 1 atm enters a 40-cm-square duct as in Fig., Using the "displacement thickness" concept, estimate (a) the mean velocity and (b) the mean pressure in the core of the flow at the position $x = 3$ m. (c) What is the average gradient, in Pa/m, in this section?



SOLUTION

For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Using laminar boundary-layer theory, compute the displacement thickness at x = 3 m:

$$\mathrm{Re}_x = \frac{\rho U x}{\mu} = \frac{1.2(2)(3)}{1.8\mathrm{E}{-}5} = 4\mathrm{E}5 \ \text{(laminar)}, \quad \delta^* = \frac{1.721x}{\mathrm{Re}_x^{1/2}} = \frac{1.721(3)}{(4\mathrm{E}5)^{1/2}} \approx 0.0082 \text{ m}$$

$$\text{Then, by continuity,} \quad V_{exit} = V\left(\frac{L_o}{L_o - 2\delta^*}\right)^2 = (2.0)\left(\frac{0.4}{0.4 - 0.0164}\right)^2$$

$$\approx \mathbf{2.175\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

The pressure change in the (frictionless) core flow is estimated from Bernoulli's equation:

$$p_{exit} + \frac{\rho}{2}V_{exit}^2 = p_o + \frac{\rho}{2}V_o^2, \quad \text{or:} \quad p_{exit} + \frac{1.2}{2}(2.175)^2 = 1 \text{ atm} + \frac{1.2}{2}(2.0)^2$$

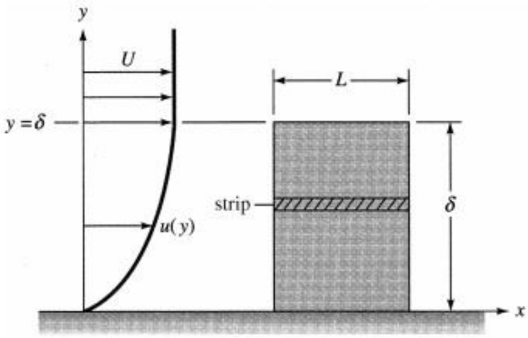
$$\text{Solve for} \quad p\big|_{x=3m} = 1 \text{ atm} - 0.44 \text{ Pa} = \mathbf{-0.44\ Pa}\ (\textit{gage}) \quad \textit{Ans.}\ \text{(b)}$$

The average pressure gradient is $\Delta p/x = (-0.44 \text{ Pa}/3.0 \text{ m}) \approx \mathbf{-0.15\ Pa/m}$ *Ans.* (c)

6-14.

A flat plate of length L and height δ is placed at a wall and is parallel to an approaching boundary layer, as in Fig. . Assume that the flow over the plate is fully turbulent and that the approaching flow is a one-seventh-power law

$$u(y) = U_o\left(\frac{y}{\delta}\right)^{1/7}$$



Using strip theory, derive a formula for the drag coefficient of this plate. Compare this result with the drag of the same plate immersed in a uniform stream U_o.

SOLUTION

For a 'strip' of plate dy high and L long, subjected to flow $u(y)$, the force is

$$dF = C_D\frac{\rho}{2}u^2(L\ dy)(2\ \text{sides}), \quad \text{where } C_D \approx \frac{0.031}{(\rho uL/\mu)^{1/7}}, \quad \text{combine into dF and integrate:}$$

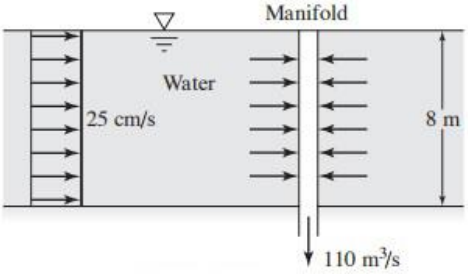
$$dF = 0.031\rho\nu^{1/7}L^{6/7}u^{13/7}dy, \quad \text{or} \quad F = 0.031\rho\nu^{1/7}L^{6/7}\int_0^{\delta}\left[U_o(y/\delta)^{1/7}\right]^{13/7}dy$$

The result is $\mathbf{F = 0.031(49/62)\rho\nu^{1/7}L^{6/7}U_O^{13/7}\delta}$ *Ans.*

This drag is (49/62), or 79%, of the force on the same plate immersed in a uniform stream.

6-15.

A coastal power plant takes in cooling water through a vertical perforated manifold, as in Fig. . The total volume flow intake is 110 m^3/s. Currents of 25 cm/s flow past the manifold, as shown. Estimate (a) how far downstream and (b) how far normal to the paper the effects of the intake are felt in the ambient 8-m-deep waters.



SOLUTION

The sink strength *m* leads to the desired dimensions. The distance downstream from the sink is *a* and the distance normal to the paper is *pa* (see Fig. 8.6):

$$m = \frac{Q}{2\pi\, b} = \frac{110 m^3/s}{2\pi(8m)} = 2.19\frac{m^2}{s}$$

$$a = \frac{m}{U_\infty} = \frac{2.19 m^2/s}{0.25 m/s} = 8.75\, m \qquad Ans.(a)$$

$$Width = \pi\, a = \pi(8.75m) = 27.5\, m \text{ on each side} \qquad Ans.(b)$$